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ABSTRACT: In this paper, we look at corporate fiscal policies set by two competing regions in an environment where firms are heterogenous regarding to their mobility costs. We show that if regions are allow to tax domestic and foreign capital at different rates, they will offer a preferential treatment to foreign firms, even if mobility costs are symmetrically distributed across regions. Preventing such type of preferential treatment raises revenues for both regions, unless there exist a high density of firms with low moving costs. Because preferential tax treatment promotes firms movement for fiscal raisons, such tax regime always generates more social loss due to unnecessary delocalization. We also investigate the effect of heterogeneity among regions.

JEL Codes: H73, H77

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1 Introduction

A controversial issue in the study of tax competition is whether it is desirable for countries or regions to agree not to provide preferential treatment to different forms of capital. The common view is that without such restrictions, countries will aggressively compete for capital that is relatively mobile across different locations, resulting in taxes that are far below their efficient level. By eliminating such preferential treatment, no capital will be taxed at very low rates, because doing so would sacrifice too much tax revenue from the relatively immobile capital. But this solution is not without cost: in an attempt to attract mobile capital, governments can be expected to reduce the common tax rate below the tax at which relatively immobile would be taxed in the preferential case. In an important paper, Keen (2001) analyzes this tradeoff using a model with two tax bases that exhibit different degrees of mobility and finds that governments raise more revenue when the more mobile tax base gets preferential treatment. On the other hand, Haupt and Peters (2005) show that preferential regimes lead to lower tax revenues by simply adding a preference for investing in the home country to the Keen (2001) framework. Surprisingly, this minor assumption changes completely the desirability of one regime versus the other.

Attacking the issue from a slightly different angle, Janeba and Peters (1999) show that the elimination of preferential treatment leads to higher total tax revenues in a context where there exist multiple arbitrary identifiable tax bases. Other papers have generalized and extended these results, including Wilson (2005), Konrad (2007), and Marceau, Mongrain and Wilson (2009). They all consider an environment with two tax bases that exogenously differ in their degrees of mobility. A general insight appears to be that if one of the two tax bases becomes perfectly mobile, so that the region with the lowest tax rate always attracts it, then the non-preferential regime raises more revenue. Both Keen (2001) and Haupt and Peters (2005) arguments in favor of preferential treatment are based on a model where the two tax bases exhibit finite, but different, elasticities with respect to the difference in the regions’ tax rates. They cannot analyze the case where one of the tax base elasticities approaches infinity, because pure-strategy equilibria may not exist. Wilson (2005) and Marceau, Mongrain and Wilson (2010) consider mixed-strategy equilibria, and obtain results supporting Janeba and Peters (1999). Most closely related to what we do is Janeba and Smart (2003), who offer a more balance view on discrimination, stating conditions where one regime is preferable to the other. The main difference is in the definition of
the tax bases. Instead of assuming exogenously defined tax bases, we look at tax bases in the original context of Keen (2001) and Haupt and Peters (2005), where origin is the only identifiable characteristic. Note that Bucovetsky and Haufler (2008) also offer a balance view, but require an additional lair of decision making relative to the organizational structure of the firms.

In the current paper, we present a case for the non-preferential or the preferential regime that does not require either tax base elasticity to be large, not to have intrinsic exogenous differences. Moreover, whereas the previous literature is silent about the magnitude of the difference in tax revenue between the two regimes, our results suggest that the difference can be quite large. Indeed, replacing preferential treatment with non-preferential treatment can nearly doubles tax revenue in our basic model with symmetric regions, and moving costs uniformly distributed. Obviously, the particular distribution moving costs takes influences the magnitude by which one regime dominates the other. If there are enough firms with low moving costs preferential regimes can generate higher tax revenue like in Keen (2001).

These results are based on a different view of the meaning of preferential tax treatment. One view in this literature is that governments distinguish between different types of capital, or firms according to their mobility characteristics. But other literatures on optimal taxation in an open economy emphasize the difficulties involved in making such distinctions. Preferential treatment must be based on observable characteristics of firms that may be only loosely associated with mobility differences.\(^1\) Thus, preferential tax regimes often consist of the foreign-owned portion of a tax base being taxed at a lower rate than the domestic-owned portion, a behavior that is also labeled “discrimination.” Some countries – e.g. Canada and the US – have signed mutually advantageous tax treaties, which would be jeopardized if one or the other actor were to start discriminating. And the prohibition of the asymmetric treatment of foreign and domestic firms has been included in treaties in the EU and the OECD. Both the OECD and the EU are active in trying to reduce the extent of discrimination among their members.\(^2\)

\(^1\)Hong and Smart (2010) assume that all firms must face the same statutory tax rates, and they analyze the use of tax havens to achieve desirable differences in effective marginal tax rates. Hagen, Osmundsen, and Schjelderup (1998) work with a model where a firm’s mobility is related to the size of its investment, in which case it is optimal to impose a nonlinear tax on investment.

\(^2\)On this, see OECD (1998).
We model this distinction between foreign and domestic firms by considering a two-region world in which each region initially possesses a stock of “domestic firms” which must incur a cost to relocate to the other region, whereas the “foreign firm” that it seeks to attract are the other region’s domestic firms. Although both foreign and domestic firms face the same distribution of moving costs, this does not mean that the tax elasticities of the foreign and domestic tax bases are the same, because these elasticities depend on how many firms of each type are located in each regions. In fact, the foreign tax base elasticity goes to infinity as the excess of the domestic tax rate over the foreign tax rate goes to zero. In particular, there is a large incentive to lower the tax on foreign firms enough to attract at least a small number of foreign firms since before the tax rates are lowered there are no current foreign firms in the region. We thus generate a preferential tax system where foreign firms are taxed more lightly than domestic firms with no intrinsic difference in elasticities. Our model also allows us to identify another negative impact associated with preferential tax regimes. With a unique tax rate on domestic and foreign firms, movement of firms is unidirectional; firms move from the low tax region to the high tax region. In fact, with symmetric regions, no firms move in equilibrium. In a preferential tax regime however, both regions set a lower tax rate on foreign capital, and so there exist a bidirectional movement of firms, even in the symmetric case. Since firms move only for fiscal reasons, preferential regimes lead to more wasteful reallocation costs.

The plan of this paper is as follows. First, we consider a simple example where all firms face a single moving cost. This example quickly produces a huge difference between the revenue raised under the non-preferential and preferential regimes. Next, we develop the basic model, in which moving costs are uniformly distributed. Then we add the zero-moving-cost firms to the analysis. Finally, we consider the dynamic model.

2 Basic Model

The economy contains two regions, indexed by $j \in \{1, 2\}$. In each region, there are $N_j$ domestic firms, with $N_1 \geq N_2$. For simplicity, we assume that $N_1 = 1$, and we define $n \in [0, 1]$ as an index of size heterogeneity between the two regions, where $n = N_2/N_1$. If both regions have the same size, then $n = 1$; heterogeneity grows as $n$ diminishes. All firms have the possibility of moving, but face different moving
costs \( c \). Moving costs are distributed between zero and one according to a cumulative distribution function \( F(c) \), and the corresponding density distribution function \( f(c) \).

Each firm generates \( \gamma \geq 1 \) of before-tax profits for its owners, regardless of where it is located. Profits are taxed where they are earned. Thus, any movement of firms from one region to another is based on tax considerations and will therefore result in the expenditure of socially wasteful moving costs. To guarantee a unique and pure strategy equilibrium, we assume that \( f'(c)/f(c) \in [1/\gamma, 1/\gamma] \). This is to say that the distribution function displays no large peaks or valleys. As a source of example, we will often use the function \( F(c) = \frac{(c+a)^{\alpha}a^\alpha}{(1+a)^{\alpha}-a^\alpha} \), with \( \alpha > 0 \) and \( a \in [0, 1] \). First, note that \( F(0) = 0 \) and \( F(1) = 1 \). Also, \( f(c) = \alpha \frac{(c+a)^{\alpha-1}}{(1+a)^{\alpha}-a^\alpha} \), which is increasing when \( \alpha > 1 \), and decreasing when \( \alpha < 1 \). The uniform distribution is a special case where \( \alpha = 1 \). Moreover, the condition stated above implies that \( \alpha \in [1-a/\gamma, 1+a/\gamma] \).

The general timing of events is as follows. First, regions choose their tax rates, with the goal of maximizing tax revenue, and all firms draw a moving cost. Then, firms chose whether to move or remain in their initial location. Finally, taxes are collected.

### 2.1 Non- Preferential Regime

Under a non-preferential regime, each region \( i \in \{1, 2\} \) sets a unique tax rate \( t_i \) for all firms, regardless of whether the firm is already in the region (domestic firms), or just moved to the region (foreign firms). For any given \( t_1 > t_2 \), a firm in region 1 stays in region 1 as long as \( [1-t_1] \gamma \geq [1-t_2] \gamma - c \). Thus, all firms with \( c > [t_1-t_2] \gamma \) stay in region 1, and so the tax revenue from all domestic firms is defined as \( [1-F(\gamma [t_1-t_2])] \gamma t_1 \). Since no firms move from region 2 to region 1 when \( t_1 > t_2 \), this quantity represents the total tax revenue for region 1. If \( t_1 \leq t_2 \), tax revenue from domestic firms is given by \( \gamma t_1 \). Firms in region 2 move to region 1 whenever \( c < [t_2-t_1] \gamma \), generating tax revenue of \( n F(\gamma [t_2-t_1]) \gamma t_1 \) for region 1. Total tax revenue in region 1, denoted \( R_1(t_1, t_2) \), is given by:

\[
R_1(t_1, t_2) = \begin{cases} 
1-F(\gamma [t_1-t_2]) \gamma t_1 & \text{if } t_1 \geq t_2; \\
\gamma t_1 + n F(\gamma [t_2-t_1]) \gamma t_1 & \text{if } t_1 < t_2.
\end{cases}
\] (1)
The best-response function $t_1(t_2)$ is defined as:

$$
t_1(t_2) = \begin{cases} 
\frac{1-F(t_1-t_2)}{\gamma f(t_1-t_2)} & \text{if } t_1 \geq t_2; \\
\frac{t_1 n f(t_1-t_2)}{\gamma n f(t_1-t_2)} & \text{if } t_1 < t_2.
\end{cases}
$$

\textbf{Lemma 1:} Whenever $f'(c)/f(c) \in [-1/\gamma, 1/\gamma]$, the best response function for region 1 is monotonically upward sloping, with a slope less than one.

\textbf{Proof of Lemma 1:} The slope of the best response function is given by:

$$
\frac{\partial t_1(t_2)}{\partial t_2} = \begin{cases} 
\frac{f(t_1-t_2)+\gamma t_1 f'(t_1-t_2)}{2f(t_1-t_2)+\gamma t_1 f'(t_1-t_2)} & \text{if } t_1 \geq t_2; \\
\frac{f(t_2-t_1)-\gamma t_1 f'(t_2-t_1)}{2f(t_2-t_1)-\gamma t_1 f'(t_2-t_1)} & \text{if } t_1 < t_2.
\end{cases}
$$

To guarantee that the second order conditions are satisfied, the denominator must be positive. For any value of $t_1 \geq t_2$, the best response function is then positively sloped, only if the numerator is also positive. Note that any condition guaranteeing that the numerator is positive also implies that the denominator will be positive. Consequently, it must be the case that $t_1 f'(c)/f(c) > -1/\gamma$. This condition is always satisfied whenever $f'(c) > 0$. When $f'(c) < 0$, it is sufficient (but not necessary) that the condition been satisfied at $t_1 = 1$. Consequently, a sufficient condition for the reaction function to be positively sloped is that $f'(c)/f(c) > -1/\gamma$. Similarly, we can show that the reaction function is positively sloped for the case where $t_1 < t_2$ when $f'(c)/f(c) < 1/\gamma$. It easy to see that if the reaction function is upward sloping, then the slope is less than one. QED

For future use, denote by $\epsilon_i = -\frac{\partial B_i}{B_i \partial t_i}$ the elasticity of the tax base $B_i$, with respect to tax $t_i$. The tax base elasticity $\epsilon_1$ for region 1 is given by:

$$
\epsilon_1 = \begin{cases} 
\gamma t_1 \frac{f(t_1-t_2)}{1-F(t_1-t_2)} & \text{if } t_1 \geq t_2; \\
\gamma t_1 \frac{n f(t_2-t_1)}{1+n f(t_2-t_1)} & \text{if } t_1 < t_2.
\end{cases}
$$

Tax revenues from region 2 are defined in a similar way:

$$
R_2(t_1, t_2) = \begin{cases} 
n [1-F(t_2-t_1)] \gamma t_2 & \text{if } t_2 \geq t_1; \\
n \gamma t_2 + F(t_1-t_2) \gamma t_2 & \text{if } t_2 < t_1.
\end{cases}
$$
The best-response function for region 2 is given by:

\[
t_2(t_1) = \begin{cases} 
\frac{1-F(\gamma[t_2-t_1])}{\gamma f(\gamma[t_2-t_1])} & \text{if } t_2 \geq t_1; \\
\frac{n+F(\gamma[t_1-t_2])}{\gamma f(\gamma[t_1-t_2])} & \text{if } t_2 < t_1.
\end{cases}
\]  

(6)

Lemma 1 also applies for region 2’s best response function, and so the function has a positive slope whenever \(f'(c)/f(c) \in [-1/\gamma, 1/\gamma]\). We can again define the tax base elasticity \(\epsilon_2\) for region 2 as:

\[
\epsilon_2 = \begin{cases} 
\gamma t_2 \frac{f(\gamma[t_2-t_1])}{1-F(\gamma[t_2-t_1])} & \text{if } t_2 \geq t_1; \\
\gamma t_2 \frac{f(\gamma[t_1-t_2])}{n+F(\gamma[t_1-t_2])} & \text{if } t_2 < t_1.
\end{cases}
\]  

(7)

**Proposition 1:** Under a non-preferential regime, if \(n = 1\), there exists a unique Nash equilibrium where \(t_1 = t_2 = \frac{1}{\gamma f(0)}\).

**Proof of Proposition 1:** Given Lemma 1 both reaction functions must cross only once. Solving equations (6) and (7) reveals that \(t_1 = t_2\). QED

**Corollary to Proposition 1:** Under a non-preferential regime with moving costs uniformly distributed, whenever \(n = 1\), there exist an unique Nash equilibrium where \(t_1 = t_2 = \frac{1}{\gamma}\).

An increase in either \(\gamma\) or \(f(0)\) lead to a reduction in tax rates. Starting from \(t_1 = t_2\), it is easy to see that the tax base elasticities are increasing in both \(\gamma\) and \(f(0)\). As \(\gamma\) increases, firms become more mobile, since the average moving cost becomes small relative to the fiscal benefit of moving. Similarly, if \(f(0)\) is large, “many firms are ready to move at no cost. Note also that the elasticity of the tax base with respect to tax rate is the same regardless of whether it is created by attracting more new firms or retaining more existing firms. Figure 1 illustrates the case where \(F(c)\) is a uniform distribution.

Whenever \(n < 1\), the smallest region is more aggressive at lowering its tax rate. This confirms many similar results in the literature on tax competition like in Bucovetsky (1991). In our context however, small is defined as the region with the least amount of domestic firms, or less immobile capital following the interpretation of Marceau, Mongrain and Wilson (2010). The next proposition relates the equilibrium tax rates to differences in regional size.
Proposition 2: Under a non-preferential regime, whenever \( n < 1 \), there exist a unique Nash equilibrium where \( t_1(n) > t_2(n) \).

Proof of Proposition 2: First, we prove by contradiction that in any equilibrium, whenever \( n < 1 \), it must be the case that \( t_1 > t_2 \). Imagine that a combination of tax rates \( t_2 > t_1 \) solves both best response functions. From (??) and (??), we could then show that:

\[
\frac{1 - F(\gamma [t_2 - t_1])}{t_2} = \frac{1 + nF(\gamma [t_2 - t_1])}{nt_1}.
\]

Re-writing the equation above, we would get that:

\[
\frac{t_1}{t_2} = \frac{1 + nF(\gamma [t_2 - t_1])}{n - nF(\gamma [t_2 - t_1])} > 1.
\]

Since \( n < 1 \), it must be the case that the left hand side of the equation above is greater than 1. Consequently, we must have that \( t_1 > t_2 \), which is a contradiction. The same contradiction does not apply for value of \( t_1 > t_2 \). Moreover, when \( t_2 = 0 \), then \( t_1(0) > 0 \). We also know that \( \frac{\partial t_1(t_2)}{\partial t_2} < 1 < \frac{\partial t_2(t_1)}{\partial t_1} \), so consequently there exist a unique Nash equilibrium where \( t_1(n) > t_2(n) \). QED

Corollary to Proposition 2: Under a non-preferential regime with moving costs uniformly distributed, whenever \( n < 1 \), there exist a unique Nash equilibrium where
\[ t_1(n) = \frac{2+n}{3\gamma} \text{ and } t_2(n) = \frac{1+2n}{3\gamma}. \]

Figure 2: Asymmetric equilibrium with uniform distribution when \( n < 1 \)

See Figure 2 for a representation of the asymmetric Nash equilibrium when moving costs are uniformly distributed. We can explain the differences in tax rates, by looking at the elasticities for both regions. From the first order conditions, we can easily show that in equilibrium, both elasticities are equalized \( (\epsilon_1 = \epsilon_2) \). When size heterogeneity increases (lower \( n \)), the small region faces a more elastic tax base, and so sets a much lower tax rate. Having fewer domestic firms gives the small region a strategic advantage. Attracting new firms has a bigger proportional impact on tax revenue for the small region. Moreover, as shown in Proposition 3, more heterogeneity in size leads to lower tax rate for both regions. More heterogeneity makes the smaller region more aggressive, and since tax rates are strategic complements, both regions set lower tax rates. As heterogeneity increases the difference in tax rates, as defined as \( \Delta_t = t_1 - t_2 \), also grows. This implies that more heterogeneity also leads to more movement of firms.

**Proposition 3:** More heterogeneity in size (lower \( n \)) leads to a lower tax rate for both Regions. Moreover, the difference in tax rates \( \Delta_t = t_1 - t_2 \) increases as heterogeneity increases.

**Proof of Proposition 3:** For any value of \( t_1 > t_2 \), the first order condition for region
1 is independent of \( n \). Using the best response function for region 2 – equation (\( ?? \)) –, we can show that

\[
\frac{\partial t_2(t_1)}{\partial n} = \frac{1}{2f(\gamma(t_2-t_1)) + \gamma t_2 f'(\gamma(t_2-t_1))} > 0 \text{ if } t_1 \geq t_2.
\] (8)

Since \( t_1(t_2) \) is increasing in \( t_2 \), it implies that both \( t_1 \) and \( t_2 \) are both increasing with \( n \). We can define \( \Delta_t = t_1 - t_2 \) as

\[
\Delta_t = \frac{1 - F(\gamma \Delta_t)}{\gamma f(\gamma \Delta_t)} - \frac{n + F(\gamma \Delta_t)}{\gamma f(\gamma \Delta_t)}
\] (9)

Comparative static reveals that:

\[
\frac{\partial \Delta_t}{\partial n} = -\frac{1/\gamma}{3f(\gamma \Delta_t) + \gamma \Delta_t f'(\gamma \Delta_t)}.
\] (10)

The expression above is always negative when \( f'(c)/f(c) \in [-1/\gamma, 1/\gamma] \), so a decrease in \( n \) leads to an increase in \( \Delta_t \). QED

We now look at tax revenue. With symmetric regions \( (n = 1) \), total tax revenue for both regions are given by \( R_i = 1/f(0) \) for the general case, and simply \( R_i = 1 \) for the uniform specification. When regions are of different sizes, total tax revenues are given by:

\[
R_1 = \left[ \frac{1 - F(\gamma[1-t_2])}{f(\gamma[1-t_2])} \right]^2, \quad R_2 = \left[ \frac{n + F(\gamma[1-t_2])}{f(\gamma[1-t_2])} \right]^2.
\]

With uniform moving costs, tax revenue are given by \( R_1 = (2 + n)^2/9 \) and \( R_2 = (1 + 2n)^2/9 \). More size heterogeneity has two important effects on tax revenue. The first one, which we label as the tax rate effect, represent the fact that more size heterogeneity pushes both regions to set lower tax rates. As the small region gets smaller, it faces higher tax elasticity and become more aggressive. The large region has no other choices that to also lower its own tax rate. The second force at play is what we refer to as the mobility effect. Firms seeking lower tax rates influence both regions tax bases by moving. This effect is also referred in the literature as tax base importing/exporting. As \( n \) decreases, \( \Delta_t = t_1 - t_2 \) increases, and so more firms are moving from region 1 to region 2. The large region always suffers from an increase in size heterogeneity. More heterogeneity implies lower tax rates, but also implies that more firms move away from region 1. A lower tax rate and a smaller base definitively lead to lower tax revenue. For the smaller region however, the two effects are working in opposite directions. More heterogeneity leads to an increase in region
2’s tax base. An increase in tax revenue can then happen if \( \partial \Delta_t / \partial n \) is sufficiently negative. To get meaningful conditions on whether heterogeneity lead to more or less tax revenue for region 2, we must however, look at an alternative measure of tax revenue. Since a reduction in \( n \) implies a smaller amount of domestic firms in region 2 \( (n = 1/n) \), changing \( n \) is consequently a bias exercise because it reduces the total tax base available in the economy.

To better understand the negative impact of tax competition, we constructed a series of measures that relates actual tax revenue to the potential tax revenue. Denote by \( r_i = R_i/N_i \gamma \), the ratio of tax revenue to potential tax revenue in region \( i \). For the uniform distribution, we get that \( r_1 = (2 + n)^2 / 9 \gamma \) and \( r_2 = (1 + 2n)^2 / 9n \gamma \). As expected, \( r_1 \) increases with a \( n \). The same applies for the general case. On the other hand, more heterogeneity in size (smaller \( n \)) has an ambiguous effect on relative tax revenues for the small region. For the uniform case, an increase in heterogeneity leads to lower tax revenue relative to potential tax revenue if \( n > 1/2 \), but leads to higher relative tax revenue when \( n < 1/2 \). A similar, but less elegant, condition also exist form the general case.

It is also useful to look at total tax revenue for both regions relative to total potential tax revenue in the economy; this measure is denoted by \( r = (R_1 + R_2) / (1 + n) \gamma \). In a non-preferential tax regime, \( r = \frac{(2+n)^2 + (1+2n)^2}{9(1+n) \gamma} \) for the uniform case. An increase in heterogeneity (smaller \( n \)) leads to lower total tax revenue. The tax rate effect leads to less tax revenue as both regions set lower tax rate. The mobility effects simply cancel each other out, as the loss for region 1 is a gain region 2. However, there exist a third effect when we look at total tax revenue, which we call the composition effect. Because firms move from region 1 to region 2, and because region 2s tax rate is lower than region 1s tax rate, total tax revenue decreases.

The last measure we look at is total moving costs relative to potential tax revenue. In a non-preferential regime, firms only move in one direction. The difference in tax payments given by Proposition 2 determines the share of region 1s domestic firms that choose to move to region 2. A total of \( F(\gamma|t_1 - t_2|) \) firms choose to move, which adds up to \( (1-n)/3 \) in the uniform case. These firms incur wasteful moving costs without any gains in productivity. The sum of moving cost generated is \( \int_{0}^{\frac{1-n}{3}} cf(c) dc \) which equal to \( \frac{(1-n)^2}{18} \) in the uniform case. Total moving costs relative to potential tax revenue for the uniform case is then given by \( \rho = (1-n)^2 / 18(1+n) \gamma \). With more heterogeneity (lower \( n \)), more firms move, and so moving cost increases.
2.2 Preferential Regime

Under a preferential tax regime, each region $i$ taxes its existing domestic firms and newly arrived foreign firms at different rates, $t_i$ for domestic firms and $\tau_i$ for the foreign firms. When $t_1 > \tau_2$, a firm in region 1 will stay in region 1 if $[1 - \tau_1] \gamma \geq [1 - t_2] \gamma - c$, or $c > [\tau_1 - t_2] \gamma$. As a result, the tax revenue obtained from all domestic firms is given by $[1 - F(\gamma[t_1 - \tau_2])] \gamma t_1$. On the other hand, if $t_1 \leq \tau_2$, all domestic firms stay, yielding tax revenue equal to $\gamma t_1$. In addition, firms in region 2 will move to region 1 whenever $c < [t_2 - \tau_1] \gamma$, generating additional tax revenue of $nF(\gamma[t_2 - \tau_1]) \gamma \tau_1$ for region 1. Total tax revenue in region 1, $R_1(t_1, \tau_1, t_2, \tau_2)$, is given by:

$$R_1 = \begin{cases} 
[1 - F(\gamma[t_1 - \tau_2])] \gamma t_1 & \text{if } t_1 > \tau_2 \text{ and } \tau_1 \geq t_2; \\
[1 - F(\gamma[t_1 - \tau_2])] \gamma t_1 + nF(\gamma[t_2 - \tau_1]) \gamma \tau_1 & \text{if } t_1 > \tau_2 \text{ and } \tau_1 < t_2; \\
\gamma t_1 & \text{if } t_1 \leq \tau_2 \text{ and } \tau_1 \geq t_2; \\
\gamma t_1 + nF(\gamma[t_2 - \tau_1]) \gamma \tau_1 & \text{if } t_1 \leq \tau_2 \text{ and } \tau_1 < t_2. 
\end{cases}$$

(11)

The best-response functions for $t_1$ and $\tau_1$ are given by:

$$t_1(\tau_2) = \frac{1 - F(\gamma[t_1 - \tau_2])}{\gamma F(\gamma[t_1 - \tau_2])} \frac{\gamma t_1}{\gamma \tau_1} \text{ if } t_1 > \tau_2;$$
$$t_1(\tau_2) = \frac{\gamma t_1}{\gamma \tau_1} \frac{\gamma t_1}{\gamma \tau_1} \text{ if } t_1 \leq \tau_2 \leq 1;$$
$$t_1(\tau_2) = 1 \text{ if } t_1 \leq \tau_2 > 1. \quad (12)$$

$$\tau_1(t_2) = \frac{F(\gamma[t_2 - \tau_1])}{\gamma F(\gamma[t_2 - \tau_1])} \text{ if } \tau_1 < t_2;$$
$$\tau_1(t_2) = 0 \text{ if } \tau_1 \geq t_2. \quad (13)$$

We start by looking at the domestic tax rate $t_1$. For any given $t_1 > \tau_2$, region 1 loses some domestic firms to region 2. Consequently, region 1 faces an elastic tax bases, and so the reaction function – first line of equation (12) – directly comes from the first-order condition on $t_1$. When $t_1 \leq \tau_2$, region 1’s tax base becomes perfectly inelastic as region 1 retains all its firms. Region 1 would like to set 100% tax rate on domestic firms. However, this cannot be done unless region’s 2 tax rate is also equal to 100%. Consequently, $t_1$ must equal $\tau_2$. Similarly, if $\tau_1 < t_2$, some firms move from region 2 to region 1, and so region 1 faces a foreign elastic tax base. This generates the reaction function given by equation (13). When $\tau_1 \geq t_2$, no firms move to region 1, so any $\tau_1$ yields zero tax revenue.
Denote by $\epsilon_d^i(t_i) = -(t_i/B^d_i) \left( \partial B^d_i / \partial t_i \right)$ the elasticity of the domestic tax base $B^d_i$ with respect to tax $t_i$, and $\epsilon_f^i(\tau_i) = -(\tau_i/B^f_i) \left( \partial B^f_i / \partial \tau_i \right)$ the elasticity of the foreign tax base $B^f_i$ with respect to tax $\tau_i$. The tax base elasticities for region 1 are given by:

$$\epsilon_d^1(t_1) = \gamma t_1 \frac{f(\gamma[t_1-\tau_2])}{1-F(\gamma[t_1-\tau_2])} \text{ if } t_1 > \tau_2;$$

$$0 \text{ if } t_1 < \tau_2. \quad (14)$$

$$\epsilon_f^1(\tau_1) = \gamma \tau_1 \frac{f(\gamma[t_2-\tau_1])}{F(\gamma[t_2-\tau_1])} \text{ if } \tau_1 < t_2;$$

$$0 \text{ if } \tau_1 > t_2. \quad (15)$$

We now look at region 2’s tax revenues maximization problem. Tax revenues for region 2 are given by:

$$R_2 = n \left[ 1 - F\left( \gamma[t_2 - \tau_1] \right) \right] \gamma t_2 \text{ if } t_2 > \tau_1 \text{ and } \tau_2 \geq t_1;$$

$$n \left[ 1 - F\left( \gamma[t_2 - \tau_1] \right) \right] \gamma t_2 + F\left( \gamma[t_1 - \tau_2] \right) \gamma \tau_2 \text{ if } t_2 > \tau_1 \text{ and } \tau_2 < t_1;$$

$$n \gamma t_2 \text{ if } t_2 \leq \tau_1 \text{ and } \tau_2 \geq t_1;$$

$$n \gamma t_2 + F\left( \gamma[t_1 - \tau_2] \right) \gamma \tau_2 \text{ if } t_2 \leq \tau_1 \text{ and } \tau_2 < t_1. \quad (16)$$

The best-response functions $t_2(\tau_1)$ and $\tau_2(t_1)$ are given by:

$$t_2(\tau_1) = \frac{1-F(\gamma[t_2-\tau_1])}{\gamma f(\gamma[t_2-\tau_1])} \text{ if } t_2 > \tau_1;$$

$$\tau_1 \text{ if } t_2 \leq \tau_1 \leq 1;$$

$$1 \text{ if } t_2 \leq \tau_1 > 1; \quad (17)$$

$$\tau_2(t_1) = \frac{F(\gamma[t_1-\tau_2])}{\gamma f(\gamma[t_1-\tau_2])} \text{ if } \tau_2 < t_1;$$

$$0 \text{ if } \tau_2 \geq t_1. \quad (18)$$

As we can see from these best response functions, there exists an equilibrium set of taxes $\{t_1, \tau_2\}$ arising from both regions competing for firms located in region 1. Similarly, there exists an equilibrium set of taxes $\{\tau_1, t_2\}$ resulting from the competition for firms located in region 2. The next proposition identifies these taxes.
Proposition 4: Under a preferential regime, there exists a unique Nash equilibrium where domestic tax rates $t_1 = t_2$ are greater than the foreign tax rates $\tau_1 = \tau_2$.

Proof of Proposition 4: Given the first order conditions no solution can be found for value of $t_1 < \tau_2$ or $t_2 < \tau_1$. Consequently, the domestic tax rate for region 1, and the foreign tax rate for region 2 are such that $t_1 > \tau_2$, and the slopes of the best response function are given by:

$$\frac{\partial t_1(t_2)}{\partial \tau_2} = \frac{f(\gamma|t_1(\tau_2) - \tau_2|) + \gamma t_1(\tau_2)f'(\gamma|t_1(\tau_2) - \tau_2|)}{2f(\gamma|t_1(\tau_2) - \tau_2|) + \gamma t_1(\tau_2)f'(\gamma|t_1(\tau_2) - \tau_2|)}.$$

(19)

$$\frac{\partial \tau_2(t_1)}{\partial t_1} = \frac{f(\gamma|t_2(\tau_2) - \tau_2|) - \gamma \tau_2 f'(\gamma|t_1(\tau_2) - \tau_2|)}{2f(\gamma|t_1(\tau_2) - \tau_2|) - \gamma \tau_2 f'(\gamma|t_1(\tau_2) - \tau_2|)}.$$

(20)

The best response for domestic tax rate by region 1 is upward sloping with a slope less than one for the same condition $f'(c)/f(c) \in [-1/\gamma, 1/\gamma]$ stated in the non-preferential regime section. QED

Corollary to Proposition 4: Under a preferential regime with moving costs uniformly distributed, there exists a unique Nash equilibrium where $t_1 = t_2 = \frac{2}{3\gamma}$ and $\tau_1 = \tau_2 = \frac{1}{3\gamma}$.

![Figure 3: Preferential taxes $t_1$ and $\tau_2$ for uniform moving costs.](image)
As we see in Figure 3, a region always sets a lower tax rate on foreign firms compared to domestic firms, independently of \( N_1 \) and \( N_2 \). Preferential tax treatment is always used to attract foreign firms. To better understand this result, we can examine the tax-base elasticities. In equilibrium, the domestic and foreign tax base elasticities are positive, and are equalized. Imagine that \( \tau_1 \) was to be smaller than \( t_2 \), but only by a very small amount. Region 1 would then attract almost no firms from region 2. Reducing \( \tau_1 \) further would then change its foreign tax base by a large proportion. This implies a large foreign tax base elasticity \( \epsilon_f(\tau_1) \). On the other hand, region 2 would lose few domestic firms. Increasing its tax rate on domestic firms would only reduce its domestic tax base by a small proportion. This implies a small domestic tax base elasticity \( \epsilon_d(t_2) \). As the gap in tax rate increases, both elasticities converges to the point where there are equal, and \( \tau_i \leq t_i \).

Two important differences arise under the preferential tax treatment. First, heterogeneity does not matter anymore. Both regions fight independently for the tax base generated by firm in region 1 and for the tax base generated by firms in region 2. Competition for domestic tax rate \( t_1 \) and for foreign tax rate \( \tau_2 \) only depends on the size of the domestic tax base in region 1. Competition for \( t_2 \) and \( \tau_1 \) only depends on the number of domestic firms in region 2. Second, firms move in both directions. Firms in region 1 with low moving cost seek low foreign tax rate in region 2, and at the same time, firms in region 2 with low moving cost seek low foreign tax rate in region 1. For the uniform case, a total of \( N_i/3 \) firms move from each region \( i \), creating a sum of moving costs equal to \( (1 + n) \int_0^{1/3} cdc = \frac{1+n}{18} \). Total moving cost relative to potential tax revenue is given by \( \rho = 1/18\gamma \). With this bi-directional movement of firms, the preferential regime always generates more wasteful moving cost than the non-preferential regime. We will now look at other differences between the two regimes by looking at tax revenue for both regions:

\[
R_1 = \frac{\left[1 - F(\gamma[t_1 - \tau_2])\right]^2}{f(\gamma[t_1 - \tau_2])} + n \frac{\left[F(\gamma[t_2 - \tau_1])\right]^2}{f(\gamma[t_2 - \tau_1])}, \quad R_2 = n \frac{\left[1 - F(\gamma[t_2 - \tau_1])\right]^2}{f(\gamma[t_2 - \tau_1])} + \frac{\left[F(\gamma[t_1 - \tau_2])\right]^2}{f(\gamma[t_1 - \tau_2])}.
\]

For the uniform case, a simple calculations yield tax revenues of \( R_1 = (4 + n)/9 \) for region 1 and \( R_2 = (4n + 1)/9 \) for region 2. Two obvious observations can be made under the uniform case. First, tax revenues are decreasing with heterogeneity. The reason is simple, smaller \( n \) implies less taxable income because there are less firms in the economy. All other effects are absent, since tax rates are independent
of heterogeneity. In fact the same applies for any general distribution of moving costs. As we already know, however, only looking at change in total revenue can be misleading because of the effect changes on \( n \) as on potential taxable income. If we instead look at tax revenues relative to potential tax revenue \( r_1 = R_1/\gamma \) and \( r_2 = R_2/n\gamma \), we find that \( r_1 \) is decreasing with heterogeneity, while \( r_2 \) is increasing with heterogeneity. With fewer firms in region 2 \( (n < 1) \), region 1 losses more firms than it gains, but region 2 attracts more firms that it looses. Total tax revenue divided by potential tax revenue \( r = [R_1 + R_2]/(1 + n)\gamma \) is obviously independent of heterogeneity because tax rate are independent of heterogeneity.

The second important observation in the uniform case is that tax revenues are lower as compare to the non-preferential regime for all value of \( n > 0 \). If the two regions have the same size \( (n = 1) \) for example, each collected four-ninths of the revenue obtained under the non-preferential regime, representing a substantial revenue loss. This however, is not always the case when we move away from the uniform distribution of moving cost. In fact, a preferential tax regime may generate larger tax revenues as stated by Proposition 5 bellow.

**Proposition 5:** With homogenous region \( (n = 1) \), the preferential tax regime generates more tax revenues if the distribution of moving costs features a sufficiently decreasing density distribution function. More precisely, if and only if:

\[
\frac{f(\gamma [t - \tau])}{f(0)} < [1 - F(\gamma [t - \tau])]^2 + F(\gamma [t - \tau])^2.
\]

A sufficient condition for the preferential tax regime to generate more tax revenues is that:

\[
\frac{f(0)}{f(\gamma [t - \tau])} \geq 2.
\]

**Proof of Proposition 5:** The tax revenues for region 1 under a preferential tax regime are given by:

\[
R_1 = \left[ \frac{[1 - F(\gamma [t_1 - \tau_2])]^2 + nF(\gamma [t_1 - \tau_2])^2}{f(\gamma [t_1 - \tau_2])} \right]. \tag{21}
\]

Tax revenues \( R_1 \) are larger than tax revenues under a non-preferential tax regime \( R_1 = 1/f(0) \), only if the condition stated in Proposition 5 is satisfied. Since the right hand side of the condition is less one, it must be the case that \( f(\gamma [t_1 - \tau_2]) < f(0) \), and so the density distribution function must be sufficiently decreasing. Since,
\[ [1 - F(\gamma [t_1 - \tau_2])]^2 + F(\gamma [t_1 - \tau_2])^2 \geq \frac{1}{2}, \text{ a sufficient sufficient condition is that } \frac{f(0)}{f(\gamma [t_1 - \tau_2])} \geq 2. \text{ QED} \]

Note that the uniform distribution of moving costs definitively does not satisfy this condition. This explains why the preferential tax regime generates less tax revenues for this distribution of moving costs. With the distribution function \( F(c) = \frac{(c+a)^\alpha a^\alpha}{(1+a)^\alpha - a^\alpha} \) we specified earlier, any value of \( \alpha \) such that \( a^{\alpha-1} > 2\gamma \) leads to a superiority of the preferential regime in term of generating tax revenues. Since \( a < 1 \), a low value of \( \alpha \) is required, meaning that the density function must be sufficiently decreasing. When the distribution of moving costs features a decreasing density, many firms are easily attracted, even for small differences in tax rates between two regions. Home bias behaviours when making investment decisions as described in Haupt and Peters (2005) would correspond to a distribution function which does not satisfied this condition, as few firms would be willing to move under this assumption. Many other reasons can account for distribution functions that would either satisfied or not satisfy the condition stated in Proposition 5. Consequently, this model can nest both Keen (2001) and Haupt and Peters (2005) models.

Lastly, we can comment on the effect of size heterogeneity on the relative merit of both types of tax regimes. For the large region, an increase in size heterogeneity favors the preferential tax regime in term of generating tax revenues. More heterogeneity reduces tax revenues in a non-preferential regime because of both the tax rate and the mobility effects. However, since tax rates are independent of \( n \) in the preferential regime, such regime is more likely to be preferred by the larger region for high degree of heterogeneity. Intuitively, the strategic advantage the small region has against the large region in the non-preferential regime comes from the different in size. This strategic advantage disappears under the preferential regime. The same is not always true for the small region. Because the mobility effect favors the small region in a preferential tax regime, more heterogeneity can make the non-preferential tax regime more attractive for the small region. Intuitively, the small region likes to compete with a unique tax rate against a large region who is reluctant to lower it tax rate due to the size of its domestic tax base. Overall, since mobility effects are a zero sum game, more heterogeneity globally favors the preferential regime.
3 Perfectly Mobile Firms

We now extend our model to include a mass of firms with zero moving cost in both regions. Moving costs are now distributed between zero and one in the following way: in each region, a proportion $\lambda$ of firms face a zero moving cost, and a proportion $1-\lambda$ have a moving cost distributed between zero and one according to a cumulative distribution function $F(c)$, and a density $f(c)$. As a source of example, we will use the same functional form proposed in the previous section. We will look at both tax regimes the same way we did in the last section.

3.1 Non- Preferential Regime

Under a non-preferential regime, each region chooses a unique tax rate on all firms, regardless of whether the firms are already located in the region, or just moved in to the region. For any given $t_1 > t_2$, a firm in region 1 stays in region 1 as long as $(1-t_1)\gamma \geq (1-t_2)\gamma - c$; this implies that all firms with zero moving cost also moves to region 2. All firms with $c > (t_1 - t_2)\gamma$ will stay in region 1. The tax revenue from all domestic firms is $(1-\lambda)[1-F(\gamma[t_1-t_2])]\gamma t_1$. Since no firm moves from region 2 to region 1, it represents total tax revenues for region 1. If $t_1 \leq t_2$, tax revenue from domestic firms is simply $\gamma t_1$. Firms in region 2 move to region 1 whenever $c < (t_2 - t_1)\gamma$, including all firms with a zero moving cost. This generates tax revenues of $\lambda n \gamma t_1 + (1-\lambda)n F(\gamma[t_2-t_1])\gamma t_1$ for region 1. Total tax revenues in region 1, $R_1(t_1, t_2)$, are given by

$$R_1(t_1, t_2) = \begin{cases} (1-\lambda)[1-F(\gamma[t_1-t_2])]\gamma t_1 & \text{if } t_1 > t_2 \\ \gamma t_1 + \lambda n \gamma t_1 + (1-\lambda)n F(\gamma[t_2-t_1])\gamma t_1 & \text{if } t_1 \leq t_2 \end{cases}$$

(22)

The best-response function for region 1 can be defined by:

$$t_1(t_2) = \begin{cases} \frac{1-F(\gamma[t_1-t_2])}{\gamma f(\gamma[t_1-t_2])} & \text{if } t_1 > t_2 \\ \frac{(1+\lambda n)/n(1-\lambda)+F(\gamma[t_2-t_1])}{\gamma f(\gamma[t_2-t_1])} & \text{if } t_1 < t_2 \\ t_1 = t_2 - \iota & \text{if } t_1 = t_2, \end{cases}$$

(23)
where \( \iota > 0 \) is as small as possible. Similarly, tax revenues from region 2 are defined as:

\[
R_2(t_1, t_2) = \begin{cases} 
(1 - \lambda)n[1 - F(\gamma[t_2 - t_1])] \gamma t_2 & \text{if } t_2 > t_1 \\
n \gamma t_2 + \lambda \gamma t_2 + (1 - \lambda)F(\gamma[t_1 - t_2]) \gamma t_2 & \text{if } t_2 \leq t_1
\end{cases}
\]  

The best-response function for region 2 is given by:

\[
t_2(t_1) = \begin{cases} 
\frac{1 - F(\gamma[t_2 - t_1])}{\gamma f(\gamma[t_2 - t_1])} & \text{if } t_2 > t_1 \\
\frac{(n + \lambda)(1 - \lambda + F(\gamma[t_1 - t_2]))}{\gamma f(\gamma[t_1 - t_2])} & \text{if } t_2 < t_1 \\
t_2 = t_1 - \iota & \text{if } t_2 = t_1
\end{cases}
\]  

Both reaction functions are once again non-decreasing when \( f'(c)/f(c) \in [1/\gamma, 1/\gamma] \).

One important difference when some firms have a zero moving cost is that a pure strategy Nash equilibrium no longer exists in some cases. For example, when \( t_1 = t_2 \), both regions would benefit from lowering their own tax rate by a small amount. If a region was to do so, it would have a marginal impact on the domestic tax base, but the region would instantly attract a proportion \( \lambda \) for the foreign firms. For a pure strategy Nash equilibrium to exist, \( N_1 \) must be sufficiently greater than \( N_2 \). The next proposition describes its properties.

**Proposition 6:** Under a non-preferential regime, there exist a unique pure strategy Nash equilibrium where \( t_1 > t_2 \) if and only if \( n < 1 - 2[\lambda + (1 - \lambda)F(\theta[t_1 - t_2])] \). It is sufficient that \( n < 1 - 2\lambda \) for this condition to be satisfied.

**Proof of Proposition 6:** When \( n = 1 \), there exist no pure strategy equilibrium. Consequently, \( n \) must be smaller than one for a pure strategy equilibrium to exist. As in Proposition 2, we can show that when \( n < 1 \), it is impossible to have \( t_1 < t_2 \).

The solution to both reaction functions where \( t_1 > t_2 \) is only possible if \( n < 1 - 2[\lambda + (1 - \lambda)F(\theta[t_1 - t_2])] \).

**Corollary to Proposition 6:** Under a non-preferential regime with moving costs uniformly distributed, there exist a unique pure strategy Nash equilibrium where \( t_1 = \frac{2 - \lambda + n}{3\gamma(1 - \lambda)} \) and \( t_2 = \frac{1 + \lambda + 2n}{3\gamma(1 - \lambda)} \) if and only if \( n < 1 - 2\lambda \).
A pure strategy Nash equilibrium only exists when there is enough difference in sizes between the two regions, and when there are few perfectly-mobile firms. Heterogeneity in size is needed to get sufficient differences in tax rates. With a large difference in tax rates, region 1 losses a significant amount of tax revenue from its domestic firms when it undercut region 2’s tax rate. A low number of perfectly-mobile firms is also needed. With few perfectly-mobile firms, region 1 enjoys a small increase in tax revenue when it undercut region 2s tax rate. If the stated condition is not satisfied, there will be no pure strategy equilibrium, but there will be some mixed strategy Nash equilibria similar to the ones found in Marceau, Mongrain and Wilson (2010).

Heterogeneity in size has the same effect as in the last section. More heterogeneous regions leads to lower tax rates for both regions because it makes the small region more aggressive, and tax rate are strategic complement. It is very interesting to see that more firms with a zero tax moving cost leads to higher tax rates for both regions. Having more firms with a zero moving cost acts as a disciplining device for the small region. In any pure strategy equilibrium, the small region always attracts all perfectly mobile firms. This implies that the tax base elasticity for the small region is lower when there are more perfectly mobile firms, leading to lower tax rates.

Assuming that the condition for the existence of a the pure strategy equilibrium is satisfied, tax revenues for the two regions are given by $R_1 = (2 - \lambda + n)^2 / 9(1 - \lambda)$ and $R_2 = (1 + \lambda + 2n)^2 / 9(1 - \lambda)$ for the uniform case, and by

$$R_1 = (1 - \lambda) \left[ \frac{1 - F(\gamma(t_1 - t_2))}{F(\gamma(t_1 - t_2))} \right]^2, \quad R_2 = (1 - \lambda) \left[ \frac{(n+\lambda)/(1-\lambda)+F(\gamma(t_1-t_2))}{f(\gamma(t_1-t_2))} \right]^2,$$

for the general case. When moving costs are uniformly distributed, tax revenues relative to potential total tax revenues in both regions are given by $r_1 = (2 - \lambda + n)^2 / 9(1 - \lambda)\gamma$ and $r_2 = (1 + \lambda + 2n)^2 / 9(1 - \lambda)n\gamma$. Economy wide tax revenues relative to total potential tax revenues is given by $r = [(2 - \lambda + n)^2 + (1 + \lambda + 2n)^2] / 9(1 - \lambda)(1 + n)\gamma$. Finally, total moving costs relative to potential revenue is given by $\rho = (1 - n)^2 / 18(1 - \lambda)(1 + n)\gamma$.

The small region always gains tax revenue when $\lambda$ increases ($r_2$ is increasing in $\lambda$). With more perfectly mobile firms, tax rates are higher, and the small region attracts more firms. For the large region, having more perfectly mobile firm has an ambiguous effect. Tax revenue increases because of the higher tax rates, but at the same time the region losses more firms. With the uniform distribution, tax revenue are increasing with $\lambda$, but this is not a general result. Total tax revenue may increases
or decrease with \( \lambda \). Higher tax rates stimulate tax revenues. At the same time, more firms move from region 1 with a higher tax rate to region 2 with the smaller tax rates. This movement of firms lowers tax revenue. In the uniform case, the effect of having higher tax rates dominates. Adding more firms with a zero moving cost does not directly change total moving costs since those moving costs are nil, but it reduces the tax gap. A smaller tax gap implies that fewer firms with a positive moving cost are actually moving.

### 3.2 Preferential Regime

Under a preferential regime, we define \( t_i \) and \( \tau_i \) as region \( i \)'s tax rate on domestic and on foreign firms, respectively. When \( t_1 > \tau_2 \), a firm in region 1 stays in region 1 if \((1 - \tau_1)\gamma \geq (1 - t_2)\gamma - c\). Thus, only the firms with \( c > (\tau_1 - t_2)\gamma \) stay in region 1. Tax revenues from all domestic firms are given by \((1 - \lambda)[1 - F(\gamma[t_1 - \tau_2])]\gamma t_1\). If \( t_1 \leq \tau_2 \), then all domestic firms stay in region 1, and tax revenues from those firms are simply \( \gamma t_1 \). Firms in region 2 move to region 1 whenever \( c < (t_2 - \tau_1)\gamma \), generating tax revenues of \( \lambda n \gamma \tau_1 + (1 - \lambda)n[F(\gamma[t_2 - \tau_1])\gamma \tau_1] \) for region 1. Total tax revenues in region 1, \( R_1(t_1, \tau_1, t_2, \tau_2) \), are given by:

\[
R_1 = \begin{cases} 
(1 - \lambda)[1 - F(\gamma[t_1 - \tau_2])]\gamma t_1 & \text{if } t_1 > \tau_2 \text{ & } \tau_1 \geq t_2 \\
(1 - \lambda)[1 - F(\gamma[t_1 - \tau_2])]\gamma t_1 + \lambda n \gamma \tau_1 + (1 - \lambda)n[F(\gamma[t_2 - \tau_1])\gamma \tau_1] & \text{if } t_1 > \tau_2 \text{ & } \tau_1 < t_2 \\
\gamma t_1 + \lambda n \gamma \tau_1 + (1 - \lambda)n[F(\gamma[t_2 - \tau_1])\gamma \tau_1] & \text{if } t_1 \leq \tau_2 \text{ & } \tau_1 \geq t_2 \\
\gamma t_1 + \lambda n \gamma \tau_1 & \text{if } t_1 \leq \tau_2 \text{ & } \tau_1 < t_2 \end{cases}
\]

The best-response functions for \( t_1(\tau_2) \) and \( \tau_1(t_1) \) are given by:

\[
t_1(\tau_2) = \begin{cases} 
\frac{1-F(\gamma[t_1 - \tau_2])}{\gamma f(\gamma[t_1 - \tau_2])} & \text{if } t_1 > \tau_2 \\
\tau_2 - \iota & \text{if } t_1 \leq \tau_2.
\end{cases}
\]

\[
\tau_1(t_2) = \begin{cases} 
\frac{\lambda/(1-\lambda)+F(\gamma[t_2 - \tau_1])}{\gamma f(\gamma[t_2 - \tau_1])} & \text{if } \tau_1 < t_2 \\
t_2 - \iota & \text{if } t_1 = t_2.
\end{cases}
\]

(26)
Tax revenues for region 2 are given by:

\[ R_2 = \begin{cases} (1 - \lambda)n [1 - F(\gamma [t_2 - \tau_1])] \gamma t_2 & \text{if } t_2 > \tau_1 \text{ and } \tau_2 \geq t_1; \\
(1 - \lambda)n [1 - F(\gamma [t_2 - \tau_1])] \gamma t_2 + \lambda \gamma \tau_2 + (1 - \lambda)F(\gamma [t_1 - \tau_2]) \gamma \tau_2 & \text{if } t_2 > \tau_1 \text{ and } \tau_2 < t_1; \\
n \gamma t_2 & \text{if } t_2 \leq \tau_1 \text{ and } \tau_2 \geq t_1; \\
n \gamma t_2 + \lambda \gamma \tau_2 + (1 - \lambda)F(\gamma [t_1 - \tau_2]) \gamma \tau_2 & \text{if } t_2 \leq \tau_1 \text{ and } \tau_2 < t_1. \end{cases} \]

The best-response functions \( t_2(\tau_1) \) and \( \tau_2(t_1) \) are given by:

\[
t_2(\tau_1) = \begin{cases} \frac{1-F(\gamma [t_2 - \tau_1])}{\gamma F(\gamma [t_2 - \tau_1])} & \text{if } t_2 > \tau_1; \\
\tau_1 & \text{if } t_2 \leq \tau_1 \leq 1; \\
1 & \text{if } t_2 \leq \tau_1 > 1; \end{cases}
\]

\[ (30) \]

\[
\tau_2(t_1) = \begin{cases} \frac{\lambda/(1-\lambda)+F(\gamma [t_1 - \tau_2])}{\gamma F(\gamma [t_1 - \tau_2])} & \text{if } \tau_2 < t_1; \\
0 & \text{if } \tau_2 \geq t_1. \end{cases}
\]

As in the previous section, there exist an equilibrium set of taxes, \( \{t_1, \tau_2\} \), resulting from the competition for firms located in region 1, and a set of taxes, \( \{\tau_1, t_2\} \), resulting from the competition for capital located in region 2. Using the best-response functions, we obtain Proposition 7.

**Proposition 7:** Under a preferential regime, there exist a unique pure strategy Nash equilibrium where \( t_1 = t_2 > \tau_1 = \tau_2 \) if and only if \( \lambda < \frac{1-2F(\gamma [t_2 - \tau_1])}{2-2F(\gamma [t_1 - \tau_2])} \).

**Proof of Proposition 7:** The proof of Proposition 7 is identical to the one from Proposition 3. However, we must ensure that \( t_i > \tau_i \). Using the first order conditions on \( t \) and \( \tau \) we can show that \( t_i > \tau_i \) if and only if \( \lambda < \frac{1-2F(\gamma [t_2 - \tau_1])}{2-2F(\gamma [t_1 - \tau_2])} \). QED

**Corollary to Proposition 7:** Under a preferential tax regime with uniformly distributed moving costs, there exist a unique pure strategy Nash equilibrium where \( t_1 = t_2 = \frac{2+\lambda/(1-\lambda)}{3\gamma} \) and \( \tau_1 = \tau_2 = \frac{1+2\lambda/(1-\lambda)}{3\gamma} \), if and only if \( \lambda < 1/2 \).

If it exists, the pure strategy equilibrium with zero moving costs firms mirrors the one described in the previous section. However, if there are too many zero moving cost
firms, so if there is too much perfectly mobile capital in other words, pure strategy equilibrium may not exist, and again only mixed strategy equilibria like in Marceau, Mongrain and Wilson (2011) would be present. As oppose to what happen under a non-preferential tax regime, where the existence of a pure strategy equilibrium requires sufficient heterogeneity between regions, heterogeneity is not needed here. What is needed for the existence of a pure strategy equilibrium is a sufficiently large tax differential so that a region who looses its perfectly mobile firms has no incentives to lower its tax rate. As we learned in the previous section, domestic firms are always taxed at a higher rate than the foreign firms in a preferential tax regime, creating the needed gap. Heterogeneity plays no role because it has no impact on tax rates.

Having more zero moving cost firms allows both regions to set higher tax rates. More zero moving cost firms lowers the foreign tax base elasticity, pushing foreign tax rate upward. Since foreign and domestic tax rates are strategic complements, domestic tax rate also increases. This is true for any distribution $F(c)$. Tax revenues for both regions are given by:

$$R_1 = (1 - \lambda) \left( \frac{[1 - F(\gamma|t_1 - \tau_2)]^2}{f(\gamma|t_1 - \tau_2)} + n \frac{\lambda/(1 - \lambda) + F(\gamma|t_2 - \tau_1)}{f(\gamma|t_2 - \tau_1)} \right),$$

$$R_2 = (1 - \lambda) \left( n \frac{[1 - F(\gamma|t_2 - \tau_1)]^2}{f(\gamma|t_2 - \tau_1)} + \frac{\lambda/(1 - \lambda) + F(\gamma|t_1 - \tau_2)}{f(\gamma|t_1 - \tau_2)} \right).$$

With uniform moving costs, tax revenues relative to potential tax revenues in both regions are given by $r_1 = [(2 - \lambda)^2 + n(1 + \lambda)^2]/9(1 - \lambda)\gamma$ and $r_2 = [n(2 - \lambda)^2 + (1 + \lambda)^2]/9(1 - \lambda)n\gamma$ respectively. Having more firms with a zero moving costs acts as a discipline device limiting tax competition, this stimulates tax revenues for both regions. With uniform moving costs, this effect dominates all other effects, and so tax revenues increases with $\lambda$. However, a countervailing force is at play. Any one firm with a zero moving cost benefits from a preferential fiscal treatment by moving, and is taxed at the lower rates $\tau < t$. This reduces tax revenue for both regions. With the general formulation of moving costs, tax revenues may increase or decrease for both regions. In the non-preferential regime, firms only moved from the large region to the small region, so the small region always benefited from higher $\lambda$. This is no longer the case with preferential tax regime.

Economy wide tax revenues relative to potential tax revenues in the uniform case are given by $r = [(2 - \lambda)^2 + (1 + \lambda)^2]/9(1 - \lambda)\gamma$. Because tax rates increase with
\( \lambda \), aggregate tax revenues tend to increase with \( \lambda \). At the same time, more firms are taxed at a preferential rate. Having more zero moving cost firms can either stimulate or limit total tax revenues. In the uniform case, it stimulates tax total revenue. The ratio of total moving costs relative to potential tax revenues is given by \( \rho = (1 - 2\lambda)^2 / 18(1 - \lambda)^2 \gamma^3 \). Adding more firms with a zero moving cost does not directly change total moving costs since those moving costs are nil, but it reduces the tax gap. A smaller tax gap implies that fewer firms with a positive moving cost are actually moving.

With uniformly distributed moving costs, the non-preferential tax regime always generates more tax revenues for both regions, as it was the case in the absence of perfectly mobile capital. With a general cost distribution function, no system systematically dominates the other. However, meaningful conditions are much harder to construct because the simplicity of the symmetric case \( (n = 1) \) can no longer be exploited.

### 4 Conclusion

Two important extensions would be worth exploring. First, we could add some perfectly-mobile firms to the model. Intuition might suggest that their existence leads to lower taxes, as regions compete more aggressively to attract them. But also, taxes could actually be higher in both the preferential and non-preferential cases. Since the smaller region would attract these mobile firms, the resulting increase in its tax base would imply a lower tax base elasticity for its foreign firms, providing it with an incentive to tax these firms at a higher rate.

A second extension would be to undertake a dynamic analysis, where each region recognizes that the new firms that it attracts this period will generate tax revenue in future periods. Under the preferential regime, this means that setting taxes on new foreign firms at low enough rates to attract them this period will enable the region to tax them at higher rates in future periods. Using the assumptions from our static model about the distribution of moving costs, the non-preferential regime may yield substantially more revenue than the preferential regime.
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