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**A MODEL OF MUSIC PIRACY WITH  
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**ABSTRACT:** Anecdotal evidence and recent empirical work suggest that music piracy has differential effects on artists depending on their popularity. Existing theoretical literature cannot explain such differential effects since it is exclusively concerned with single-firm models. We present a model with two types of artists who differ in their popularity. We assume that the costs of illegal downloads increase with the scarcity of a recording, and that scarcity is negatively related to the artist's popularity. Moreover, we allow for a second source of revenues for artists apart from CD sales. These alternative revenues depend on an artist's recognition as measured by the number of consumers who obtain his recording either by purchasing the original or downloading a copy. Our findings for the more popular artist generalize a result found by Gayer and Shy (2006) who show that piracy is beneficial to the artist when alternative revenues are important. In our model, however, this does not carry over to the less popular artist, who is often harmed by piracy even when alternative revenues are important. We conclude that piracy tends to reduce musical variety.

JEL Codes: L82, K42

Keywords: Piracy, file sharing, heterogeneous artists

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# 1 Introduction

In the course of the last two decades, the economics of information goods has become a very lively discipline. Information goods, defined broadly by Shapiro and Varian (1999, p. 3) as “anything that can be digitized,” have the particular property that they can be copied basically without quality degradation. This makes them vulnerable to copyright infringement. Music is the information good that has suffered most severely from the violation of intellectual property rights. Piracy of music has been rampant since the emergence of Internet-based file sharing networks in the late 1990s. The music industry claims that this kind of private, noncommercial piracy is threatening the creation of music at large. Musicians themselves seem to be divided over whether piracy is good or bad, as a survey of American musicians and songwriters by the Pew Institute (2004) has shown. Pop star Robbie Williams has been quoted as saying that piracy is “great” (The Economist, 2003), and several artists have released their songs for free on the Internet.

At a theoretical level, economists have been studying the welfare implications of copying for some time. The basic trade-off policymakers are facing in designing copyright legislation is between under-utilization and under-production of intellectual property (see Romer, 2002). Since information goods are largely non-rival (an individual’s consumption of the good does not affect the quantity of the good available to others), efficient consumption requires all consumers with a willingness-to-pay exceeding the (small) cost of reproduction to have access to the good. Therefore, at least in the short run, consumers almost always benefit from the availability of copies.

Given that the development of an information good is typically associated with a high fixed cost, the producer would make a loss if he set the price at marginal cost (i.e., reproduction cost). Copyright confers some market power to the producer and thereby makes market provision possible. Unauthorized reproduction, however, results in the good being only partially excludable, and thus erodes the producer’s market power. The resulting decline in profits reduces the producer’s incentive to create. This leads to a problem of underprovision. Accordingly, the most basic models, relying on self-selection of consumers in the spirit of Mussa and Rosen (1978), predict piracy to be harmful to producers, which entails in the long run also negative repercussions on consumers due to reduced incentives to create (Belleflamme, 2003; Yoon, 2002; Bae and Choi, 2006).

There are several reasons why there may actually be less of a conflict between consumptive efficiency and incentives for producers than this discussion suggests. A variety of papers have shown that it may sometimes be profitable for the firm to allow some degree of piracy.<sup>1</sup> The

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<sup>1</sup> We do not only refer to the obvious case where the costs of complete prevention are so high that producers prefer to let some consumers obtain the product for free. As noted by King and Lampe (2003, p. 272), research

first case is when producers can indirectly appropriate the consumers' rent from copying by charging a higher price to those buyers who are going to have more copies made from their originals (Liebowitz, 1985). The second case is the presence of positive network effects on the demand side. If a consumer's valuation depends on how many others are consuming the good, piracy allows the monopolist to take advantage of network effects while maintaining a high price and extracting surplus from high-valuation consumers (Conner and Rumelt, 1991; Takeyama, 1994; Shy and Thisse, 1999). The third case is sampling: since music is an experience good and tastes are heterogeneous, consumers do not know beforehand whether or not they like a particular piece of music. File sharing enables consumers to try out new musical genres and artists, which may under some conditions increase demand (Peitz and Waelbroeck, 2006b; Zhang, 2002).<sup>2</sup>

In a contribution specifically dealing with the music industry, Gayer and Shy (2006) point to a possible conflict between artists and publishers as to the desirability of unauthorized reproduction of their works. The argument is based on the observation that record sales are not the only source of income for artists (e.g., live concerts). While publishers may be harmed by piracy, artists may benefit from the increased recognition of their work that piracy brings about. The notion that piracy may increase concert revenues has some empirical support (Oberholzer-Gee and Strumpf, 2010; Mortimer, Nosko, and Sorensen, 2010).

From an empirical point of view, file sharing can provide insights regarding the impact of unauthorized copying (in particular for testing the different hypotheses put forward by the theoretical literature). So far, there is only limited support for a positive effect of piracy on demand. On the contrary, most empirical studies indicate that the record industry is being harmed (Hui and Png, 2003; Peitz and Waelbroeck, 2004; Zentner, 2006; Rob and Waldfogel, 2006; Liebowitz, 2008; Waldfogel, 2010). For example, Rob and Waldfogel (2006) and Waldfogel (2010) both estimate that the sales displacement produced by illegal downloading (the marginal effect of illegal downloads on sales) is between -0.15 and -0.3. One exception is the investigation by Oberholzer and Strumpf (2007), who find that piracy has no statistically discernible effect on album sales. Apart from this controversial result, one interesting point raised by their work is that the impact of piracy may vary across artists: some may gain while others may lose. Specifically, there is heterogeneity of the effect of downloading on sales between sales categories. The top selling quartile of albums is positively affected by downloads, while the lowest selling quartile is negatively affected.

In this paper, we present a model with two types of artists that can account for differential effects of piracy on high- and low-selling musicians. Its originality lies in the assumption of

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on law and economics tells us that this may apply to any unlawful activity provided enforcement is costly.

<sup>2</sup> For a review of the literature on piracy, see Peitz and Waelbroeck (2006a).

popularity-dependent copying costs. That is, consumers' cost of downloading depends on an artist's level of popularity (assumed exogenous). This modeling is motivated by the observation that, on average, it is much more time-consuming to find and download a recording from a little known artist than a very popular song. Following Gayer and Shy (2006), we also incorporate a feature explaining why some artists may be in favor of piracy while others oppose it by introducing an alternative source of revenues for artists. We do not, however, address the conflict of interest that may exist between artists and record labels.

Our results confirm the finding obtained in a different setting by Gayer and Shy (2006) according to which artists can be better off with piracy than without it if alternative revenues are important. But this applies in an unrestricted way only to the more popular artist. The less popular artist may still be worse off under piracy even if alternative revenues are set at their highest possible level. At first glance, this may appear counterintuitive since higher downloading costs should shield the little known artist from the adverse effects of piracy to some degree. However, the way in which the artists can benefit from piracy is by using it to their advantage. In fact, copying constitutes a cheap way of distributing an artist's recording to a greater part of his potential audience, thereby increasing the alternative revenues which are assumed to be linked to the total number of consumers who are knowledgeable about his music. If the less popular artist's popularity is in a middle range where downloading costs are not yet prohibitively high so that his music is still pirated to some extent but not enough to reach a sufficient level of non-CD sale revenues, piracy reduces his profit. From a welfare perspective, this means that piracy is detrimental at least for musical variety.

We develop a model that takes into account that piracy may affect artists in different ways depending on their level of popularity. To do this, we start from the simple framework of a monopolist selling to a continuum of consumers who self-select according to their willingness to pay. Interpreting the firm as being an artist, we extend that framework by introducing a second artist. We assume that each of the two artists sells a single good (one can think of the goods indifferently as single songs or entire albums), and that they differ in their popularity. Their levels of popularity are exogenously given, and consumers like only one of the two goods. This implies that there is no competition between artists; both are monopolists in the market for their respective product. Apart from the sales of their CDs, artists have a second source of revenues, positively related to the number of users of the good. One can think of concerts, advertising, or television appearances, for example.

One of the artists, referred to as the "star," is more popular than the other. Copies of the most popular artists' recordings are easier to obtain on file-sharing networks than those of relatively unknown artists because, in general, the number of people sharing those files is larger. We capture this property by supposing that there are higher downloading costs (for

consumers) for the less popular artist’s music. Intuitively, we would expect this modelling to result in a lesser effect of piracy on the “underground” artist, while the star should suffer more. However, this effect might be counterbalanced by the fact that opportunities to make money out of alternative sources increase with “stardom.” Piracy, by expanding the user base of a recording, leads to higher revenues from these other sources. If a star’s music is both more demanded and easier to download and is therefore copied more, we should expect that the star, while losing more in terms of CD sales, also benefits more from the increased dissemination of his recording than the less popular artist. In the formal analysis that follows, we examine the relative strength of these two effects and determine which conditions determine the respective impact of piracy on the two artists.

### **The model vis-à-vis the literature**

The general self-selection setup of the model draws on Yoon (2002). There are also similarities with other models in the literature. We now discuss briefly such common features and elaborate on what distinguishes the current model from the existing literature.

First, like in Zhang (2002), we assume that there are two artists: a star and an underground artist. However, whereas Zhang allows for competition between the two artists who in his case produce horizontally differentiated but (imperfectly) substitutable goods (the artists being located at the ends of the classic Hotelling line), we assume that the two goods are no substitutes so that demands are independent. This means that, for reasons exogenous to the model (tastes), consumers are exclusively drawn to one style of music and do not derive any utility from consuming the other (this is, of course, an extreme assumption). Moreover, “stardom” is not defined in terms of the financial capacity of the label supporting the artist (as in Zhang), but rather in terms of the proportion of the population who prefer an artist’s music to the other’s. Also in Alcalá and González-Maestre (2010), the two types of artist compete.

Second, we follow Gayer and Shy (2006) in introducing a second source of revenues for artists. Gayer and Shy, who model a conflict of interest between artists and labels, leave the decision of how to price the CD solely to the record company which is assumed to ignore the artist’s interest in setting the price. The artist gets a share of the label’s profit. By contrast, we consider only a single entity which maximizes its total profit taking into account all the artist’s revenues. This can be seen as a special case of Gayer and Shy’s approach where all the share of the profit goes to the artist and where the artist takes the pricing decision.

Taking a closer look at the pricing decision, it is clear that both the assumption that the record company sets the price without regard to the implications for the artist and the alternative assumption that the artist sets the price are extreme cases. If we accept that there

is at least some degree of competition between record companies on the “market for artists,” record companies cannot altogether ignore the artists’ interests. If there is sufficiently strong competition for signing promising artists, we may actually converge to the case where the record companies set the price of CDs as if they were the artist.

Alcalá and González-Maestre (2010) use an OLG model to endogenize the number of stars. They incorporate promotion costs that can be reduced by using piracy as a promotion device. We disregard the promotion component that adds, as the authors show, an incentive to allow piracy, and we focus instead on another transmission channel: the presence of popularity-dependent copying costs. That is, we allow the costs that consumers incur when downloading a song from a file-sharing network to vary across artists depending on their popularity. Specifically, since the songs of little known artists are in relatively scarce supply, they are costlier to download than stars’ music.

The remainder of this paper is structured as follows. In Section 2, we present the model. In Section 3, we derive the artists’ pricing decision. In Section 4, we examine the welfare effects of piracy, the emphasis being on long-term incentives to create. Finally, Section 5 concludes.

## 2 Model setup

There are two artists  $i$ : a popular artist (“star,” denoted by the subscript  $s$ ), and a less popular artist (“underground,” denoted by the subscript  $u$ ), producing products that are sufficiently horizontally differentiated for the cross-price elasticity of the demand for each product to be zero (their products are neither substitutes nor complements). Both of them are monopolists and their production technology is represented by the affine cost function

$$C_i(q) = cq + F_i \text{ for } q > 0, \text{ and } C_i(0) = 0, \quad (1)$$

where  $q$  is the quantity of reproductions of the recording (CDs),  $c$  is a constant per-unit cost which is the same for both artists, and  $F_i$  is the fixed cost of creating the recording (which may differ between the artists).

There is a mass 1 of consumers, a proportion  $\alpha$  of which appreciate (only) the star’s music, while the remaining  $(1 - \alpha)$  like (only) the less-known artist’s works, with  $\frac{1}{2} < \alpha < 1$ . Consumers differ in their valuation for music denoted  $\theta_i$ , with  $i = s, u$ , where the index  $s$  represents those consumers who prefer the star and  $u$  those preferring the underground artist. Both types of consumers have valuations uniformly distributed on  $[0, 1]$ .<sup>3</sup>

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<sup>3</sup> A more general formulation would consist in letting valuations be distributed on  $[0, \bar{\theta}_i]$ . This would allow for the possibility that the top valuation for the star may be different from that for the less-popular artist, i.e.

Consumers have unit demand for the artists' product. They have two ways to obtain the product: they can either buy the original at a price  $p_i$ , or download a copy on a file-sharing network. The consumers' cost of copying depends on the scarcity of the good, i.e., the star's music is less costly to copy than the unknown artist's music. This is because it is easier to find popular artists' recordings on file-sharing networks than very rare works. In particular, the cost may include the opportunity cost of time spent searching for and downloading the file. Given that copying of most musical recordings is illegal, the cost may also include the expected cost of detection by law enforcement authorities (Crampes and Laffont, 2002), although it is not clear whether this would differ between the two artists. Denoting  $d_i$  the cost of copying artist  $i$ , we assume  $d_u > d_s$ , i.e., copying the less popular musician is costlier than copying the star. Therefore, the utility of a consumer with valuation  $\theta_i$  is given by

$$U_{\theta_i} = \begin{cases} \theta_i - p_i & \text{if she buys the original} \\ \beta\theta_i - d_i & \text{if she copies} \\ 0 & \text{otherwise.} \end{cases}$$

The parameter  $\beta < 1$  represents the quality of the copy relative to that of the original. Presumably  $\beta$  is close to one. In fact, improvements in compression technology have made differences in sound quality quite small, although, of course, there remains some quality degradation due to lacking cover, song lyrics and other material included with the original of the recording.<sup>4</sup>

Artists have two sources of income: sales of their recordings, and revenues from various sources such as concerts, merchandizing, licensing, advertising, or television appearances, to name just a few. We assume that revenues other than CD sales depend positively on the artists' recognition as measured by the number of agents who consume their music (regardless of whether they bought or copied it). Moreover, there are increasing returns with respect to the number of users: marginal revenue from the alternative sources is increasing in the total number of distributed recordings. This reflects the fact that a small number of highly popular musicians get the bulk of lucrative advertising contracts and television appearances. Also, there are likely to be increasing returns to scale for live performances, and consumers are

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$\bar{\theta}_s \neq \bar{\theta}_u$ . One could make the argument that the highest valuation may be higher within consumers who like the star than within those loving the underground artist. A justification could come from the possible existence of network effects: If the willingness to pay of consumers depends positively on the total number of people who are knowledgeable about the recording, the top valuation for the star may be higher than that for the less-popular artist. However, the argument for network effects is rather weak in the case of music. Therefore, it is difficult to see why the respective top-valuation consumer's appreciation for the star should be greater than for less-known artist, absent objective differences in quality. Then, we should assume  $\bar{\theta}_s = \bar{\theta}_u = \bar{\theta}$ , and without loss of generality we can normalize  $\bar{\theta}$  to 1.

<sup>4</sup> It should also be noted that new technologies such as the Blu-ray Disc have once again introduced more of quality wedge between illegally downloaded and legally sold versions of an album.

willing to pay higher prices to see top acts. Accordingly, the revenue function of each artist takes the form

$$R(q, x) = P(q)q + \Phi(q + x), \text{ with } \Phi(0) = 0, \Phi' > 0, \Phi'' > 0, \quad (2)$$

where  $P(q)$  is inverse demand for CDs and  $\Phi(\cdot)$  is other revenues, while  $x$  is the number of copies made (so that  $x + q$  is the total number of users of the recording). For the sake of concreteness and simplicity, we suppose in the following that  $\Phi(\cdot)$  is quadratic, i.e.

$$\Phi(q + x) = \phi(q + x)^2,$$

where the parameter  $\phi > 0$  determines the importance of non-CD sale revenues in the artists' income.

Interestingly, this (quadratic) specification also arises naturally when the demand for live performances (as one particular source of alternative revenues) is explicitly modeled, as in the model of Gayer and Shy (2006). Our specification can therefore be interpreted as a reduced form of a model where the artist has a second activity whose demand depends (linearly) on the number of distributed recordings.

Given this setup, we assume that artists set the price of the recording (or equivalently, since both are monopolists, the quantity  $q_i$ ) so as to maximize their profit which we define in gross terms (before subtraction of the fixed creation cost  $F_i$ ), i.e.  $\pi_i = R_i(q_i) - cq_i$ .

As far as terminology is concerned, we should stress one important distinction. In what follows, we use the term *popularity* to refer to the (exogenous) proportion of consumers who like a given artist (i.e.,  $\alpha$  or  $1 - \alpha$ ), whereas by an artist's *recognition* we mean the total number of distributed recordings (legally sold originals plus illegally downloaded copies).

### 3 Profit maximization

#### 3.1 No piracy

Suppose first that copying is not possible, so that users only have the choice between buying the good and refraining from consuming it. Then, consumers buy if  $\theta_i - p_i \geq 0$ , otherwise, they don't consume. Hence, the demand addressed to the artists is

$$\begin{aligned} D_s(p_s) &= \alpha(1 - p_s) && \text{for the star artist, and} \\ D_u(p_u) &= (1 - \alpha)(1 - p_u) && \text{for the less popular artist.} \end{aligned}$$

We can calculate inverse demand to obtain:

$$P_s(q_s) = 1 - \frac{q_s}{\alpha},$$

$$P_u(q_u) = 1 - \frac{q_u}{1 - \alpha}.$$

Using the revenue function specified in (2), and substituting for  $P(q)$ , we obtain marginal revenue:

$$MR_s(q_s) = 1 - 2q_s \left( \frac{1}{\alpha} - \phi \right),$$

$$MR_u(q_u) = 1 - 2q_u \left( \frac{1}{1 - \alpha} - \phi \right).$$

The monopolists maximize profits by equalizing marginal revenue and marginal cost (given by  $c$ ). This yields the optimal quantities and optimal prices under the no piracy regime (indexed by the superscript 0):

$$q_s^0 = \frac{\alpha(1 - c)}{2(1 - \alpha\phi)}; p_s^0 = \frac{1 + c - 2\alpha\phi}{2(1 - \alpha\phi)}; q_u^0 = \frac{(1 - \alpha)(1 - c)}{2(1 - (1 - \alpha)\phi)}; p_u^0 = \frac{1 + c - 2(1 - \alpha)\phi}{2(1 - \alpha\phi)}. \quad (3)$$

These follow directly from the first-order conditions of the artists' maximization problem. In addition, for the second order condition to be satisfied, we need  $\phi < \frac{1}{\alpha}$  (which implies also  $\phi < \frac{1}{1 - \alpha}$ ). This restriction on  $\phi$  makes sure that marginal revenue is downward sloping for both artists. If it is not satisfied (i.e., if  $\phi$  is too large), so that marginal revenue slopes upward, the artists want to produce the highest possible quantity since any loss from CD sales is more than compensated by the gain in terms of other revenues. For both artists to produce a strictly positive quantity, we also need consumers' maximum willingness to pay to exceed marginal cost, that is  $c < 1$ .

It is instructive to compare these optimal price-quantity pairs to those that would prevail in the absence of a second source of revenues (which corresponds to  $\phi = 0$ ). In that case, prices would be  $\frac{1+c}{2}$  for both artists and thus higher than those given by (3) (accordingly, optimal quantities would be smaller). This is to be expected since non-CD sale revenues depend positively on the artists' recognition. In the absence of piracy, recognition is equivalent to the number of CDs sold. Hence, the artist finds it optimal to lower his price in order to gain recognition and benefit from increased non-CD sale revenues.

We can then also calculate the gross profit  $\pi_i^0$  of each artist  $i$ , defined as the profit before deduction of the fixed development cost. Since, in the absence of piracy,  $x = 0$ , we have

$$\pi_i^0 = R_i(q_i^0) - cq_i^0 = (P_i(q_i^0) - c)q_i^0 + \phi(q_i^0)^2, \quad (4)$$

so

$$\pi_s^0 = \frac{\alpha(1 - c)^2}{4(1 - \alpha\phi)} \quad (5)$$

$$\pi_u^0 = \frac{(1 - \alpha)(1 - c)^2}{4(1 - (1 - \alpha)\phi)} \quad (6)$$

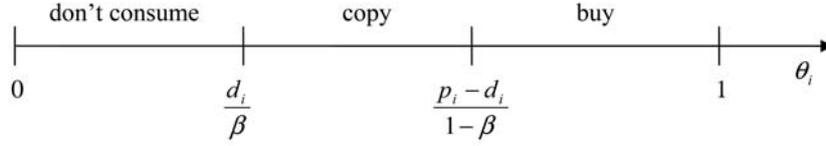


Figure 1: Self-selection of consumers

Since by assumption  $\alpha > 1/2$ , the profit of the underground artist is lower than the star's. The same applies to the quantity sold. At the same time, the price charged by the less-popular artist is higher than the star's. This is due to the convexity of the function  $\Phi(\cdot)$  which determines non-CD sale revenues. In fact, for a given price, the star faces a larger demand, and can exploit the gains from recognition more easily. More precisely, his marginal revenue from sources other than CD sales is higher than for the less popular artist. Therefore, he chooses to set his price below the level chosen by the less popular artist.

One interesting consequence of this is that the star serves a higher percentage of his potential audience than the less popular artist. This can be easily verified by taking the ratios  $q_s^0/\alpha$  and  $q_u^0/(1 - \alpha)$  which represent the part of each artist's potential audience that is actually being served.

### 3.2 Piracy

Now suppose that consumers can either buy or copy the product sold by the artists. Depending on their valuation, consumers either buy or copy or do not consume at all:

- if  $\theta_i - p_i \geq \beta\theta_i - d_i \geq 0$ , they buy the original;
- if  $\theta_i - p_i < \beta\theta_i - d_i$ , but  $\beta\theta_i - d_i \geq 0$ , they download an unauthorized reproduction;
- if  $\theta_i - p_i < 0$  and  $\beta\theta_i - d_i < 0$ , they do not consume the good.

We can then determine the threshold values of  $\theta_i$  which delimit non-consumers from copiers, and copiers from buyers. They are depicted in Figure 1 (which is valid as long as  $p_i > \frac{d_i}{\beta}$ ). Those consumers with  $\theta_i \in [0, \frac{d_i}{\beta})$  don't consume, those with  $\theta_i \in [\frac{d_i}{\beta}, \frac{p_i - d_i}{1 - \beta})$  download a copy, and those with  $\theta_i \in [\frac{p_i - d_i}{1 - \beta}, 1]$  purchase the original.

To illustrate the substitution of copies for originals that takes place, suppose that the price for music,  $p_i$ , was above  $\frac{d_i}{\beta}$  in the absence of piracy. This implies that  $p_i < \frac{p_i - d_i}{1 - \beta}$ . Suppose for a second that the artist leaves his price unchanged in the presence of file sharing. If we rewrite the consumer's utility if copying as  $\theta_i - (1 - \beta)\theta_i - d_i$ , we see that the cost of copying can be decomposed in two parts: the reproduction cost  $d_i$  which is constant across consumers,

and the degradation cost  $(1 - \beta)\theta_i$  which is proportional to the consumer's valuation. For the consumers with the lowest valuation (between 0 and  $\frac{d_i}{\beta}$ ), the possibility to copy doesn't change anything: they still don't find it worthwhile to consume the good. Similarly, the highest-valuation consumers continue to buy the original even when copies are available since their total copying costs (defined as the sum of reproduction and degradation costs) exceed the price. However, in between those two groups, there are two kinds of consumers. Some consumers who would not have consumed the good in the absence of piracy now find it worthwhile to download a copy. Some others, though, who would have purchased the original if piracy were not an option, now switch to the alternative procurement technology that consists in downloading the file. Those latter consumers, located between  $p_i$  and  $\frac{p_i - d_i}{1 - \beta}$  are the ones who substitute copies for originals.

From Figure 1, we can see that a necessary condition for piracy to take place is  $\frac{d_i}{\beta} < 1$ . Otherwise, copying is never an option for any consumer. Moreover, if  $p_i \leq \frac{d_i}{\beta}$ , there is no copying in the respective artist's market. This offers the artist a possibility to deter piracy. We come back to this below when we study the pricing decision. At this stage, the important point is the implication that we get a kinked demand curve, with the kink being located at  $(p_s; q_s) = \left(\frac{d_s}{\beta}; \alpha \left(1 - \frac{d_s}{\beta}\right)\right)$  and  $(p_u; q_u) = \left(\frac{d_u}{\beta}; (1 - \alpha) \left(1 - \frac{d_u}{\beta}\right)\right)$ , respectively. For prices below the kink, demand is the same as before. For prices above the kink, demand can be easily derived from the scheme presented in Figure 1 above.

Demand is then completely described by:

$$D_s(p_s) = \begin{cases} \alpha(1 - p_s) & \text{if } p_s \leq \frac{d_s}{\beta} \\ \alpha\left(1 - \frac{p_s - d_s}{1 - \beta}\right) & \text{if } p_s \geq \frac{d_s}{\beta} \end{cases} \quad (7)$$

$$D_u(p_u) = \begin{cases} (1 - \alpha)(1 - p_u) & \text{if } p_u \leq \frac{d_u}{\beta} \\ (1 - \alpha)\left(1 - \frac{p_u - d_u}{1 - \beta}\right) & \text{if } p_u \geq \frac{d_u}{\beta} \end{cases} \quad (8)$$

From this we compute the inverse demand:

$$p_s(q_s) = \begin{cases} 1 - \frac{q_s}{\alpha} & \text{if } q_s \geq \alpha\left(1 - \frac{d_s}{\beta}\right) \\ (1 - \beta)\left(1 - \frac{q_s}{\alpha}\right) + d_s & \text{if } q_s < \alpha\left(1 - \frac{d_s}{\beta}\right) \end{cases} \quad (9)$$

$$p_u(q_u) = \begin{cases} 1 - \frac{q_u}{1 - \alpha} & \text{if } q_u \geq (1 - \alpha)\left(1 - \frac{d_u}{\beta}\right) \\ (1 - \beta)\left(1 - \frac{q_u}{1 - \alpha}\right) + d_u & \text{if } q_u < (1 - \alpha)\left(1 - \frac{d_u}{\beta}\right) \end{cases} \quad (10)$$

Figure 2 shows the resulting kinked demand function in the case of the star. The less popular artist faces a similar demand (it suffices to replace  $\alpha$  by  $(1 - \alpha)$  and  $d_s$  by  $d_u$ ).

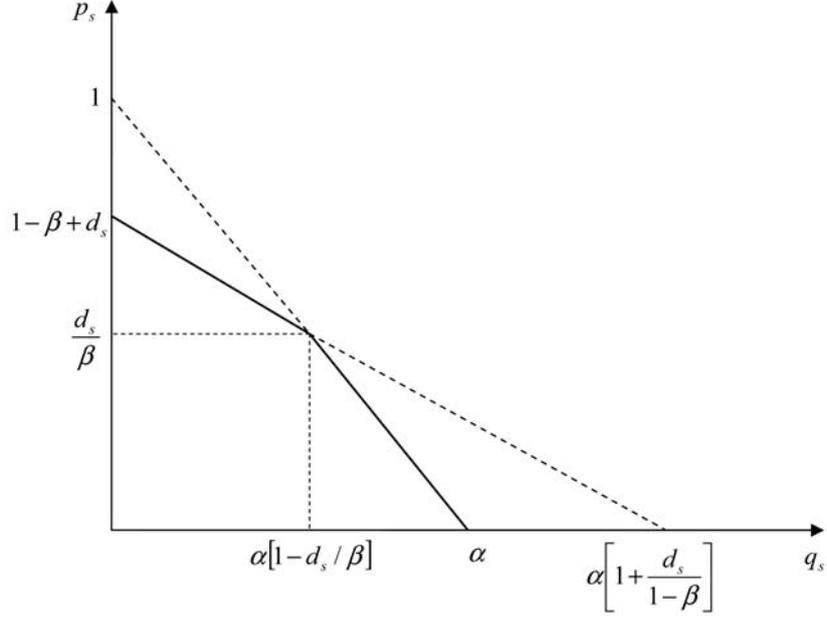


Figure 2: The kinked demand curve

To deduce marginal revenue, notice that under piracy, revenue from other sources does not solely depend on the number of recordings sold. Specifically, as long as copying takes place (that is, for prices above the limit price) the recognition of the artist is constant. This is because the number of consumers who remain out of the market (that is, who don't consume) is determined only by exogenous parameters ( $\beta$  and  $d$ ) as we can see from Figure 1.

The total number of agents consuming an artist's product is also constant and thus independent of  $q$ . It is given by  $\alpha(1 - \frac{d_s}{\beta})$  and  $(1 - \alpha)(1 - \frac{d_u}{\beta})$ , respectively. We therefore obtain the following marginal revenue functions:

$$MR_s(q_s) = \begin{cases} 1 - 2q_s(\frac{1}{\alpha} - \phi) & \text{if } q_s \geq \alpha(1 - \frac{d_s}{\beta}) \\ (1 - \beta)(1 - \frac{2q_s}{\alpha}) + d_s & \text{if } q_s < \alpha(1 - \frac{d_s}{\beta}) \end{cases} \quad (11)$$

$$MR_u(q_u) = \begin{cases} 1 - 2q_u(\frac{1}{1-\alpha} - \phi) & \text{if } q_u \geq (1 - \alpha)(1 - \frac{d_u}{\beta}) \\ (1 - \beta)(1 - \frac{2q_u}{1-\alpha}) + d_u & \text{if } q_u < (1 - \alpha)(1 - \frac{d_u}{\beta}) \end{cases} \quad (12)$$

This is illustrated in Figure 3, where the bold curve represents marginal revenue. The marginal revenue function exhibits a discontinuity at the point corresponding to the kink of the demand curve. If it were not for the parameter  $\phi$ , this would not present a problem. However, the presence of  $\phi$  may lead to technical difficulties if the discontinuity is such that marginal cost can intersect MR twice. This occurs whenever  $\phi$  is too large. In fact, the slope

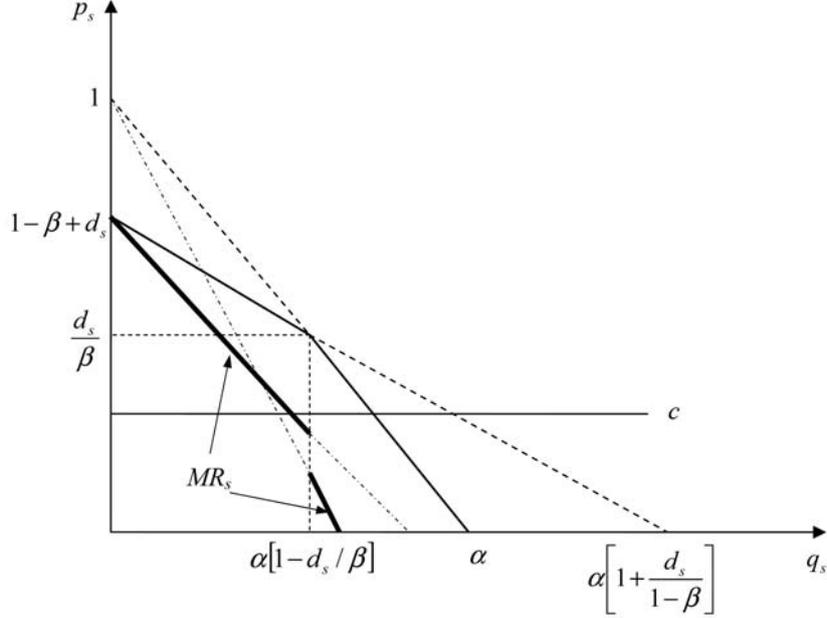


Figure 3: The marginal revenue

of the part of the curve that is located to the right of the discontinuity depends on  $\phi$ . As  $\phi$  increases, the second part of the MR curve becomes flatter.

We can impose a restriction on  $\phi$  to rule out the possibility of double intersection. As it turns out, the necessary assumption is not a strong one. In fact, we must have (for the star)

$$1 - \beta + d_s - \frac{2(1 - \beta)}{\alpha} \left( \alpha \left( 1 - \frac{d_s}{\beta} \right) \right) \geq 1 - 2 \left( \frac{1}{\alpha} - \phi \right) \left( \alpha \left( 1 - \frac{d_s}{\beta} \right) \right) \Leftrightarrow \phi \leq \frac{\beta}{2\alpha} \quad (13)$$

If the restriction is met with equality, the marginal revenue function for the star is continuous. Similarly, for the underground artist, we need  $\phi \leq \frac{\beta}{2(1 - \alpha)}$ , but this condition is implied by the condition for the star.

We assume in the following that condition (13) is satisfied. There are two reasons why this should not be considered a strong assumption. First, the numerical example in the following subsection shows that in spite of this restriction (which places an upper bound on the importance of alternative sources of revenue), the part of non-CD sale revenues in gross profits can still be very significant (especially for the star). Second, if, as we should assume,  $\beta$  is close to one, this is basically the same assumption as the one we would need in order to exclude dumping (i.e., price below marginal cost) under the no-piracy regime.<sup>5</sup> Since CDs certainly were not priced below their marginal cost of production before the emergence of

<sup>5</sup> To exclude dumping, we must have  $p_i^0 \geq c \Leftrightarrow 1 - \frac{1-c}{2(1-\alpha\phi)} \geq c \wedge 1 - \frac{1-c}{2(1-(1-\alpha)\phi)} \geq c \Leftrightarrow \phi \leq \frac{1}{2\alpha} \wedge \phi \leq \frac{1}{2(1-\alpha)}$ .

large-scale private piracy via file-sharing networks, this does not seem like much of a stretch.

We can then study the pricing decision of the artists when piracy is possible. Depending on where marginal cost intersects marginal revenue (see Figure 3), we have four possible cases. We can use the terminology introduced by Bain (1956) to classify the first three of those cases as accommodation, deterrence and blockade. Following the convention in the literature, the conditions for each of those cases can be expressed as depending on the reproduction cost  $d_i$ . For the star artist:

- if

$$d_s < \frac{\beta(1 - \beta + c)}{2 - \beta} \quad (14)$$

we are in the situation where the artist *accommodates* piracy. The optimal price is the one corresponding to the quantity that solves  $(1 - \beta) \left(1 - \frac{2q_s}{\alpha}\right) + d_s = c$ , which is  $q_s^1 = \frac{\alpha(1 - \beta + d_s - c)}{2(1 - \beta)}$ , implying that  $p_s^1 = \frac{1 - \beta + d_s - c}{2}$ . In figure 3, this corresponds to the case where marginal cost intersects the upper part of marginal revenue (left of the kink). The gross profit associated with this solution is  $\pi_s^1 = \frac{\alpha(1 - \beta + d_s - c)^2}{4(1 - \beta)} + \phi \left(\alpha \left(1 - \frac{d_s}{\beta}\right)\right)^2$ .

- If

$$\frac{\beta(1 - \beta + c)}{2 - \beta} \leq d_s < \frac{\beta(1 + c - 2\alpha\phi)}{2(1 - \alpha\phi)}, \quad (15)$$

the optimal price is the limit price  $p_s^2 = \frac{d_s}{\beta}$  which *deters* pirates. The corresponding quantity is given by  $q_s^2 = \alpha \left(1 - \frac{d_s}{\beta}\right)$ , and profit equals  $\pi_s^2 = \alpha \left(\frac{d_s}{\beta} - c\right) \left(1 - \frac{d_s}{\beta}\right) + \phi \left(\alpha \left(1 - \frac{d_s}{\beta}\right)\right)^2$ .

- If

$$d_s \geq \frac{\beta(1 + c - 2\alpha\phi)}{2(1 - \alpha\phi)}, \quad (16)$$

reproduction costs are too high for piracy to present a threat to the artist. This corresponds to the case where marginal cost intersects the lower part of marginal revenue in figure 3. Piracy is *blockaded* and the artist can charge the monopoly price under no piracy,  $p_s^0 = 1 - \frac{1 - c}{2(1 - \alpha\phi)}$ .

- For completeness, we also need to consider the case where  $c > 1 - \beta + d_s$ . In this case, the producer cannot gain a positive margin on the sales of his recordings. In the absence of other revenues, the market would break down. However, since as soon as  $q$  is strictly positive, the artist - due to piracy - reaches the same level of recognition as with any other  $q_s < \alpha \left(1 - \frac{d_s}{\beta}\right)$ , he always receives the “fixed” amount of other revenues  $\phi \left(\alpha \left(1 - \frac{d_s}{\beta}\right)\right)^2$  regardless of how many CDs he sells. Therefore, he sets the price at  $1 - \beta + d_s - \varepsilon$ , where  $\varepsilon$  is small, so that he sells an infinitesimal quantity of his

	No piracy		Piracy	
	Star	Underground	Star	Underground
Quantity sold	0.333	0.133	0.111	0.167
Price	0.5	0.6	0.375	0.458
Total profit	0.111	0.044	0.139	0.038
Non-CD sale revenues	0.056	0.009	0.134	0.017
Non-CD sale revenues/profit	50%	20%	96.7%	45.1%
Part of audience served	50%	40%	16.7%	50%
Part pirating	0	0	61.1%	5.6%
Total part consuming	50%	40%	77.8%	55.6%

Table 1: A numerical example

recording (albeit at a loss). Thanks to piracy, this enables him to reap the benefits of his recognition.

Once again, thanks to the symmetry of the problem, the same holds for the underground artist, replacing  $\alpha$  by  $(1 - \alpha)$  and  $d_s$  by  $d_u$ .

### 3.3 A numerical example

The following table presents some key results of the model by means of a numerical example. We compare the no-piracy and the piracy regimes assuming, for that example, that condition (14) is fulfilled for both artists, i.e.,  $d_i < \frac{\beta(1-\beta+c)}{2-\beta}$ , and therefore that both artists' music is pirated. Furthermore, we set the importance of non-CD sale revenues, as measured by  $\phi$ , at slightly below its highest possible value still satisfying the restriction imposed by (13). The table is based on the following parameter values:  $\alpha = 2/3, \beta = 3/4, c = 1/3, d_s = 1/6, d_u = 1/3, \phi = 1/2$ .

For the no-piracy regime, the example exhibits all the properties discussed above: the star sells a higher quantity, charges a lower price, and makes more profits than the less popular artist. His revenues from alternative sources are much higher than the less popular artist's; they account for 50 percent of his profits (compared to 20 percent for the underground artist). He also serves a larger percentage of his potential audience than the less popular artist.

The picture changes when piracy is possible. While both artists reduce their price in the face of "competition" from pirates, the number of CDs sold by the star falls by two thirds whereas the underground artist actually sells more than before. The quantity sold by the underground artist now exceeds what the star sells. This pattern is reversed when we look

at profits: The star benefits from piracy and increases his profits above the level witnessed in the absence of piracy, whereas the less popular artist loses. Whether there is some regularity to this phenomenon (the effect of piracy on the artists' profits having opposite signs for the star and the less-popular artist) is discussed in the following subsection.

Other noteworthy features include the fact that the star now gains 97 percent of his profits from activities other than CD sales, and that 61 percent of his potential customers download a copy of his product while only 17 percent buy the original (we cannot, however, exactly quantify the substitution effect since the artist changes his price in response to the availability of file sharing). For the less-known artist, only 6 percent of his potential audience pirates the good. This discrepancy is induced by the fact that the downloading cost for the less popular artist is twice as high as for the star, which reduces the interval of valuations for which copying takes place (see Figure 1). For both artists, the total percentage of consumers who obtain the product in one way or another is considerably higher than without piracy.

## 4 Welfare analysis

### 4.1 Short-term welfare

To evaluate the effect of piracy on short-term welfare, we need to compute the total surplus (net consumer surplus plus profit) under the assumption that both artists are in the market. There are several problems with this methodology. First, it may not be very meaningful to calculate surplus in the current model. In the partial equilibrium setting of the model, the source of non-CD sale revenues is not explicitly modeled: In a way, those revenues fall like manna from heaven. In reality, somebody has to pay for them. This may not be overly problematic in the case of concerts, where consumers pay directly for their tickets. However, it is much more of a concern when we think of other sources of revenue such as advertising which is sometimes considered wasteful from a social point of view. Still, we could overcome this problem by making the assumption that consumers derive zero net surplus from these other activities (i.e., that the artist extracts the entire surplus) since otherwise they would not pay for them. In other words, we could assume away socially wasteful activities.

Second, however, it is difficult enough to determine whether the artists win or lose from piracy, let alone quantify the gain or loss, as the following subsection shows. That means that actually calculating the difference in surplus between the two regimes (piracy versus no piracy) is likely to be prohibitively complicated.

Nevertheless, we can make an informed guess about the short-term welfare consequences of piracy in our model based on other results in the literature. The effect of piracy on consumer surplus is sure to be positive: in both the case of deterrence and the case of accommodation,

it gives more consumers access to the goods, and prices decrease. One result of the literature discussed in the introduction is that, in a model with linear demand as in our case, although producer profits decline, this decline is more than offset by an increase in consumer surplus, so that the total effect of piracy on social welfare is unambiguously positive (Belleflamme 2003). In our case, due to the presence of a second source of revenues for artists, it is not even sure that artists' profits decline, as we see below. Therefore, we conclude that in our model, too, the increase in consumer surplus exceeds any possible decline in profits. This means that the short-term welfare effect of piracy is positive.

## 4.2 Long-term welfare

For the long-term consequences of piracy for welfare, the important question is whether the artists' incentives to produce music remain intact. We thus have to analyze what happens to their profits under piracy. To do this, we make the assumption that the fixed cost  $F_i$  of developing the recording can differ between the artists. Such a difference might stem from different costs of writing the song, recording and mixing the initial master tape, or from different promotion and advertising expenditures, for example. Assuming that  $F_i$  can vary across artists makes the analysis of long-term welfare straightforward: If at least one artist's profit deteriorates as a result of piracy, this potentially destroys his incentives to create and must therefore be considered as detrimental from an *ex ante* efficiency perspective.

Let us assume that the condition for piracy to occur (equation (14)) holds. Piracy is beneficial to the star if and only if profit under piracy is greater than profit without piracy, that is,

$$\pi_s^1 = \frac{\alpha(1-\beta+d_s-c)^2}{4(1-\beta)} + \phi \left( \alpha \left( 1 - \frac{d_s}{\beta} \right) \right)^2 \geq \pi_s^0 = \frac{\alpha(1-c)^2}{4(1-\alpha\phi)} \quad (17a)$$

$$\Leftrightarrow \Delta\pi_s \equiv \pi_s^1 - \pi_s^0 = \frac{\alpha(1-\beta+d_s-c)^2}{4(1-\beta)} + \phi \left( \alpha \left( 1 - \frac{d_s}{\beta} \right) \right)^2 - \frac{\alpha(1-c)^2}{4(1-\alpha\phi)} \geq 0. \quad (17b)$$

Intuitively, we would expect that piracy is more beneficial to the musician the greater is  $\phi$ . As the following proposition shows, there is indeed a threshold value of  $\phi$  such that above that value, the artist is better off with piracy than without it.

**Proposition 1.** *There exists a threshold  $\hat{\phi} \in \left(0, \frac{\beta}{2\alpha}\right)$  above which piracy is beneficial for the star. This threshold is given by the solution of the (second degree) equation (17b).*

*Proof.* Equation (17b) represents a parabola in  $\phi$ , with two real roots. Notice that, for  $\phi \in \left(0, \frac{\beta}{2\alpha}\right)$ , equation (17b) is increasing in  $\phi$ :

$$\frac{\partial \Delta\pi_s}{\partial \phi} = \left( \alpha \left( 1 - \frac{d_s}{\beta} \right) \right)^2 - \left( \frac{\alpha(1-c)}{2(1-\alpha\phi)} \right)^2 > 0 \quad (18)$$

The reason for that is that the expression in the second bracket is the quantity produced in the absence of piracy which, if  $d_s < \frac{\beta(1-\beta+c)}{2-\beta}$ , is smaller than the total number of distributed recordings under piracy (sales and copies, first bracket). This is true at least as long as  $\phi \leq \frac{\beta}{2\alpha}$  (the restriction imposed on  $\phi$  for marginal revenue to be non-degenerate). So the difference in profits strictly increases with  $\phi$ .

The monotonicity of (17b) over the mentioned interval guarantees that there is one and only one intersection within the interval as long as the function assumes negatives values at the left extreme of the interval and positive values at the right one.

All we need to show is therefore that  $\Delta\pi_s$  evaluated at  $\phi = 0$  is negative (first part of the proof) while it is positive at  $\phi = \frac{\beta}{2\alpha}$  (second part of the proof). For the first part of the proof,  $\phi = 0$ , so we obtain the condition

$$(1 - \beta + d_s - c)^2 < (1 - \beta)(1 - c)^2, \quad (19)$$

which can be rewritten as

$$(\beta - d_s)^2 < 2(1 - c)(\beta - d_s) - \beta(1 - c)^2. \quad (20)$$

Given the condition  $c < 1 - \beta + d_s$  which is required for the demand to be non-negative, a sufficient condition for (20) to be satisfied is

$$(\beta - d_s)^2 < 2(1 - 1 + \beta - d_s)(\beta - d_s) - \beta(1 - c)^2$$

and thus

$$(\beta - d_s)^2 > \beta(1 - c)^2. \quad (21)$$

Using again condition  $c < 1 - \beta + d_s$  and noting that  $1 - c + d_s > d_s$ , a sufficient condition for (21) is  $(1 - c + d_s - d_s)^2 > \beta(1 - c)^2 \iff \beta < 1$ , which is always verified.

For the second part of the proof, we need to show that  $\Delta\pi_s$  is positive when evaluated at  $\phi = \frac{\beta}{2\alpha}$ , that is,

$$\frac{\alpha(1 - \beta + d_s - c)^2}{4(1 - \beta)} + \frac{\beta}{2\alpha} \left( \alpha \left( 1 - \frac{d_s}{\beta} \right) \right)^2 - \frac{\alpha(1 - c)^2}{4(1 - \frac{\beta}{2})} \geq 0. \quad (22a)$$

$$\iff \frac{(\beta^2 + 2d_s - \beta(1 + c + d_s))^2}{4\beta(1 - \beta)(2 - \beta)} \geq 0. \quad (22b)$$

This last condition is always verified, with strict inequality as long as  $(\beta^2 + 2d_s - \beta(1 + c + d)) \neq 0$ , which corresponds to saying that  $\Delta\pi_s = 0 \iff d_s = \frac{\beta(1-\beta+c)}{2-\beta}$ , that is, when  $d_s$  attains the maximum possible value for which piracy occurs.  $\square$

If revenues linked to the artists' recognition are important, piracy is beneficial to the star. We can explain this as follows. In the absence of piracy, the artist faces a trade-off between a higher margin on record sales on the one hand and higher revenues from alternative sources on the other hand, given that the latter require that he charge a lower price in order to gain recognition. Piracy gives the artist a way to increase his recognition without having to reduce his markup and therefore relaxes this constraint. In a way, it enables the artist to charge the monopoly price on his residual demand and at the same time to benefit from a high level of recognition and the associated advantages. If non-CD sale revenues are large, this effect dominates the reduction in the demand for originals that piracy entails. This extends the result obtained by Gayer and Shy (2006) to the case where the artist himself sets the price of his CDs.

If we want to make a statement about what happens to the less-known artist's profits, we have to be more precise about what determines the larger cost of piracy. Since the idea is that the costs of downloading increase with the scarcity of the artist's recordings, it seems natural to tie it either to the number of sold recordings or to the artist's popularity. Of course, in reality, the distribution of a piece of music through the different channels is a dynamic process. At the beginning, the scarcity of a copy depends mainly on the number of CDs sold and on the willingness of buyers to share the music on file-sharing networks. However, the distinctive feature of digital copying is that you can make copies of copies without losing quality. Therefore, even if the number of CDs sold is small, the cost of a download is smaller for more strongly demanded recordings since they are disseminated faster. Hence, it would appear that it is appropriate to assume that the cost of a download is linked to the proportion of the population that appreciates an artist's music. The simplest way to introduce such a relationship is to assume proportionality of downloading costs with respect to popularity. Thus, in what follows we assume that  $d_u = \frac{\alpha}{1-\alpha}d_s$ .

Then, depending on the value of  $\alpha$ , which determines the degree of (un-)popularity of the less-known artist, there are three possible cases conditional on  $d_s$  being such that the star is pirated:

1. If the star is extremely popular relative to the underground artist, so that the latter's recordings are very rare, it is prohibitively costly to copy the less-known artist. The less-known artist faces no threat from piracy. This is the case if  $\frac{\alpha d_s}{1-\alpha} \geq \frac{\beta(1+c-(1-\alpha)\phi)}{2(1-(1-\alpha)\phi)}$ .
2. If  $\frac{\beta(1+c-(1-\alpha)\phi)}{2(1-(1-\alpha)\phi)} > \frac{\alpha d_s}{1-\alpha} \geq \frac{\beta(1-\beta+c)}{2-\beta}$ , the less-known artist chooses to limit-price his product in order to deter pirates. This unambiguously hurts his profits compared to the case without piracy.

In both of those cases, only the star's music is being pirated.

3. If the level of popularity of the less-known artist exceeds a certain value determined by the condition  $\alpha < \frac{\beta(1-\beta+c)}{d_s(2-\beta)+\beta(1-\beta+c)}$ , there is piracy for both artists. Then, whether piracy is beneficial to the less-known artist depends on the sign of the difference between profits in the two regimes (piracy versus no piracy):

$$\Delta\pi_u = \frac{(1-\alpha)\left(1-\beta+\frac{\alpha d_s}{1-\alpha}-c\right)^2}{4(1-\beta)} + \phi\left((1-\alpha)\left(1-\frac{\alpha d_s}{(1-\alpha)\beta}\right)\right)^2 - \frac{(1-\alpha)(1-c)^2}{4(1-(1-\alpha)\phi)} \geq 0 \quad (23)$$

Case (2) and our numerical example above demonstrate that the star may win from piracy while the less popular artist may lose. To examine under what conditions piracy is detrimental to the less-known artist even though it is beneficial to the star, we now take a closer look at case (3). As the Proposition above shows, the star is strictly better off with piracy if  $\phi = \frac{\beta}{2\alpha}$ . Plugging this into (23), simplifying by  $(1-\alpha)$ , and rearranging, we see that the expression which determines the sign of  $\Delta\pi_u$  is a fourth-degree polynomial in  $\alpha$  (the numerator):

$$\frac{\Delta\pi_u}{1-\alpha} = \frac{\lambda_0 - \lambda_1\alpha + \lambda_2\alpha^2 - \lambda_3\alpha^3 + \lambda_4\alpha^4}{4\alpha\beta(1-\alpha)^2(1-\beta)(\alpha\beta + 2\alpha - \beta)} \quad (24)$$

where

$$\lambda_0 = 2\beta^3(\beta - 1) \quad (25a)$$

$$\lambda_1 = \beta^2(9\beta^2 + 2\beta(c + 2d_s - 3) + c^2 - 2c - 4d_s - 3) \quad (25b)$$

$$\lambda_2 = \beta(15\beta^3 + 2\beta^2(3c + 7d_s - 2) + \beta(5c^2 + 2c(d_s - 3) + 2d_s^2 - 6d_s - 11) - 2d_s(d_s + 4)) \quad (25c)$$

$$\lambda_3 = 11\beta^4 + 2\beta^3(3c + 8d_s + 1) + \beta^2(7c^2 + 2c(2d_s - 3) + 5d_s^2 + 4d_s - 13) + 4\beta d_s(c - 5) - 4d_s^2 \quad (25d)$$

$$\lambda_4 = 3\beta^4 + 2\beta^3(c + 3d_s + 1) + \beta^2(3c^2 + 2c(d_s - 1) + 3d_s^2 + 6d_s - 5) + 4\beta d_s(c + d_s - 3) - 4d_s^2. \quad (25e)$$

Due to the high degree of difficulty of the problem, we perform a numerical analysis of this expression. We conjecture that  $\Delta\pi_u$  is decreasing in  $\alpha$  and has a root between  $1/2$  and  $\frac{\beta(1-\beta+c)}{d_s(2-\beta)+\beta(1-\beta+c)}$ . Our conjecture is supported numerically for many different parameter configurations. In particular, this is the case for the parameters used in our numerical example from Section 3.3. Figure 4 shows the graph of the expression that determines the sign of  $\Delta\pi_u$  as a function of  $\alpha$  over the relevant range given by  $\left(\frac{1}{2}, \frac{\beta(1-\beta+c)}{d_s(2-\beta)+\beta(1-\beta+c)}\right)$  for the following configuration:  $\beta = 3/4, c = 1/3, d_s = 1/6, \phi = 9/16$ . This corresponds to the parameters used in the example of section (3.3), with the exception of  $\phi$  which has been set at its maximum value consistent with (13). As can be seen from Figure 4,  $\Delta\pi_u$  is positive for low values of  $\alpha$  but turns negative from the point where  $\alpha = 0.58$ .

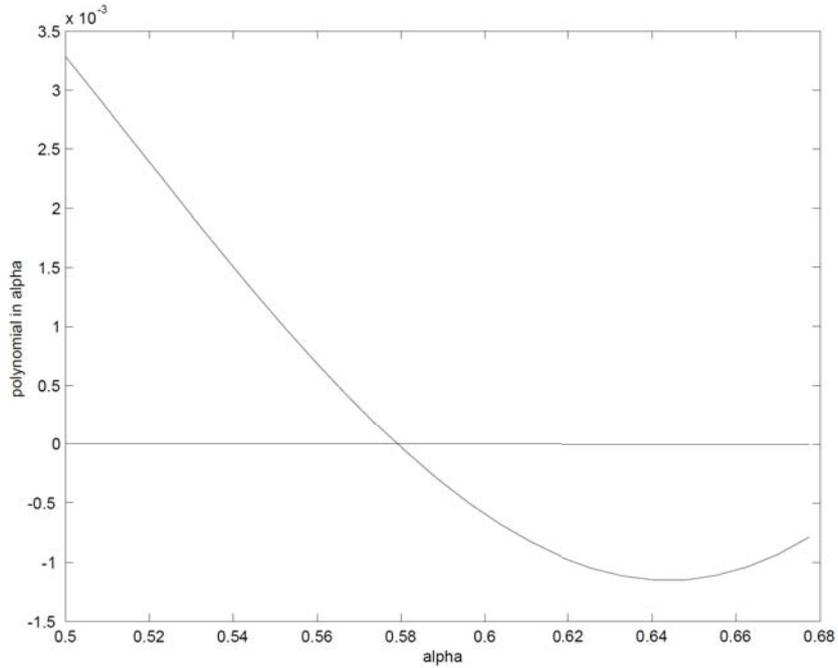


Figure 4: The difference in the underground artist's profit with and without piracy

The intuition for this result is the following. For values of  $\alpha$  close to  $1/2$ , the underground artist does not differ much in his popularity from the star. Accordingly, the cost of downloading his music is only slightly higher than for the star. He therefore benefits from the same effect that the star enjoys which, as described above, consists in getting increased recognition without having to make concessions regarding the markup on CDs. Initially, this effect outweighs the substitution of copies for originals caused by the availability of file sharing. However, as the popularity of the less-known artist decreases, the cost of downloading his music rises so that less and less consumers copy. This means that the revenues linked to his recognition fall, and although his CD sales now suffer less than the star's, the piracy-induced reduction in demand can no longer be compensated by alternative revenues. In a way, for this range of  $\alpha$ , the underground musician is caught in the middle: he is not popular enough to replace lost CD sales by revenues out of other sources, but he is too popular for piracy to be blockaded. In that case, we must conclude that piracy is bad for welfare since, by reducing the less popular artist's profits, it may keep him out of the market and therefore reduce musical variety.

In summary, we have seen that the effects of piracy depend very much on the parameters of the model. Even assuming we are in a configuration where there is piracy of (at least) the star's music, everything is contingent on  $\phi$  and  $\alpha$ . If revenues from sources other than

CD sales are important, piracy is beneficial for the star. However, it need not be for the less popular artist if he finds himself in a middle range of popularity where he can enjoy neither sufficient gains from recognition, nor shelter from pirates.

## 5 Conclusion

We have presented a simple model of music piracy with popularity-dependent copying costs. The theoretical literature is largely silent on how piracy may affect different types of artists since it is concerned almost exclusively with single-firm models. By contrast, anecdotal evidence and recent empirical work suggest that copying has differential effects on artists depending on their popularity.

We propose to deal with this issue by setting up a model with two types of artists who differ in their popularity, and by letting the cost incurred by consumers when downloading an artist's recording vary with the artist's level of popularity. More precisely, we assume that downloading costs increase with the scarcity of a recording, and that scarcity is negatively related to the artist's popularity. Moreover, we allow for a second source of revenues for artists apart from CD sales. We make the assumption that these alternative revenues are an increasing and convex function of an artist's recognition as measured by the number of consumers who obtain his recording either by purchasing the original or downloading a copy.

Our findings for the more popular artist generalize a result found in a different kind of setup by Gayer and Shy (2006) who assert that piracy is beneficial to the artist when alternative revenues are important. However, in our model this does not carry over to the less popular artist, who is found in certain cases to be harmed by piracy even when the parameter measuring the importance of alternative revenues is set at its maximum. Therefore, we conclude that piracy is bad for social welfare since it is likely to reduce musical variety. This negative result may be mitigated when piracy, through its effect on recognition, has an impact on the probability of an underground artist to become a star, as in Alcalá and González-Maestre (2010), which is likely to occur under some imperfections in the talent revelation process (see Terviö, 2009).

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## 2011

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